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NAVAL POSTGRADUATE SCHOOL
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THESIS

TURBULENCE EFFECTS ON A
GLOW DISCHARGE AS DERIVED FROM
CONTINUITY AND ENERGY CONSIDERATIONS

by

William R. Oker

September 1984

Thesis Advisor: Oscar Biblarz

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It is shown that the modification of the ambipolar diffusion coefficient and the thermal conductivity is a reasonably sufficient model to introduce the turbulence effects. It is found that the charged particle density is effectively the sole source of heating in the gaseous environment of the discharge. It is then shown that turbulence acts to supress the temperature instability introduced by the discharge streamer.

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Turbulence Effects on a Glow Discharge as Derived from Continuity and Energy Considerations

by

William R. Oker
Lieutenant Commander, United States Navy
B.S., California State Polytechnic University, 1972

Submitted in partial fulfillment of the
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LIST OF SYMBOLS

A	=	area
α	=	$\alpha_3 n_o/\alpha_2$
α_2	=	two-body recombination coefficient
α_3	=	three-body recombination coefficient
β	=	inverse turbulence parameter due to ambipolar diffusion
β_A	=	inverse turbulence parameter due to thermal conductivity
D_a	=	ambipolar diffusion coefficient
e	=	internal energy, or elementary electron charge
E, E_o	=	electric field strength, initial value
G	=	conductance
I	=	current
j	=	current density
K	=	thermal conductivity
k	=	Boltzman's constant
ℓ	=	length of streamer (or distance between electrodes)
m_e	=	electron mass
μ_e	=	electron mobility
n, n_o	=	charged particle density, initial value
$n_{\mathfrak{C}}$	=	n value on streamer centerline
$\hat{n}$	=	non-dimensionalized n
n_e	=	number of electrons
N	=	total number of particles
ν_i	=	ionization coefficient

ν_e = electron collision frequency

Ω = resistance

P = pressure

q = heat flux

r, r_o = streamer channel radius, initial value

$\hat{r}$ = non-dimensionalized radius

ρ = density

σ, σ_o = conductivity, initial value

T, T_o = temperature, initial value

$\hat{T}$ = non-dimensionalized temperature

τ_R = characteristic recombination time

TR = same as τ_R

τ_A = characteristic time for energy equation

t, t_o = time, initial value

$\hat{t}$ = non-dimensionalized time

t_A = t associated with τ_A

U = gas flow velocity

v_d = electron drift velocity

V = voltage

Z = electric field perturbation parameter for continuity equation

Z_A = "Z" for energy equation

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I wish to dedicate this thesis to my wife, Barbara. Without her constant love, support, and understanding, this work would not have been possible.

I. INTRODUCTION

If a gas such as nitrogen or argon, or a mixture of gases such as air, is subjected to an electrical potential, a sequence of phenomena take place as a result of a continuously increasing applied voltage and/or current. Initially, for a small voltage, a very weak current on the order of 0.1 picoamperes develops. This current will increase slightly with increasing voltage until all the naturally occurring charge carriers are being utilized. A current saturation condition now prevails until the applied voltage reaches about 6 kV in air at which time a further voltage rise rapidly increases the current flow to about one picoampere. This is referred to as the Townsend Discharge Region and it is due to new charge carriers being created by collisions between the highly energetic electrons (fast moving) and the neutral (slow moving) atoms. This process can have several effects on the individual gas molecules. Essentially, it can excite them, dissociate them, and/or ionize them. This ionization effect is higher for lower gas pressures since the farther the electron travels between collisions the higher will be its kinetic energy which is imparted by the acceleration produced by the applied electric field. When the electron's energy is greater than that required for ionization of the neutral gas particles the collisions will

produce additional electrons which in turn gain energy, produce collisions, and therefore, increase the total number of electrons present. This process is called an electron avalanche.

From this point, slight increases in voltage produce current increases of several orders of magnitude at which time the current becomes self sustaining. Reducing the external circuit resistance now increases the current flow and a steady glow is produced in the gas. This glow-discharge is the familiar phenomenon which occurs in neon lights and has other practical applications such as a pumping scheme for gas lasers.

Finally, when the current is increased still further, a highly ionized channel is formed which collapses the glow. If there is sufficient current carrying capacity in the external circuit a steady current is established which represents a plasma state called an arc.

A full understanding of the arcing mechanism is difficult due to the extremely short formation times involved. However, since delaying the point of transition to arcing is a highly practical problem, a method for modeling it is needed. A blend of theory and experiment is usually required. Such a model could then be used to study the effects of the arc on and by its environment.

It has been observed that this glow-to-arc transition can be delayed by utilizing gas dynamic stabilization

techniques in flowing gas systems [Ref. 1]. The purpose of this work is to establish the extent of fluid dynamic stabilization that results from the introduction of varying levels of fluid dynamic turbulence into the flow. It will be shown that this method has beneficial effects on the stability of the glow discharge process and on the temperature distribution in the gas produced by the presence of discharge streamers. For further discussion and background information relating to ionization and the breakdown processes refer to any standard reference in the field or to Wallace, R.J. [Ref. 2].

An introductory skill level in atomic physics plus fundamental proficiencies in mechanics, electrical principles and numerical analysis should provide the reader with adequate preparation for understanding this material.

The principles discussed here have many practical engineering applications. For maximum power output of an electrically pumped gas laser, such as a carbon dioxide laser, the glow discharge must be held essentially at the limit of arcing. Since turbulent flows tend to delay this glow-to-arc transition, the laser can be allowed to operate at either the highest possible power level or with a greater margin of safety at lower power levels. Another application where a stabilized high frequency electrical spark discharge could be used as a means for injecting heat into a propulsive duct (like a pulsejet) has been suggested by C.E. Tharratt [Ref. 3].

This type of a device is theoretically capable of unlimited thrust. It would also have a low specific fuel consumption and could operate outside the audible range. The possibility of nearly silent operation would be highly advantageous as an aircraft propulsion device for the future.

Two major cases will be discussed. The first case concerns the behavior of a highly transient discharge in a non-ionized gaseous environment such as the lightning discharge. This first case concerns a pre-glow discharge type phenomenon since the streamer propagates in a non-ionized medium. However, it serves to establish the effects of turbulence in a simpler situation in order to form a starting place for this research. The second case will look at discharge streamers in a partially ionized gas which might be used for the pumping mechanism of a gas laser. The method of approach to these problems will be developed in the next chapter where a mathematical model will be presented. The governing partial differential equations for the model presented will be solved by numerical integration techniques utilizing a computer. A working Fortran program is presented which can be used to investigate a wide variety of the aspects of this problem. Significant results are then presented in graphical format to demonstrate the stabilizing features of the turbulent flow.

II. ANALYSIS

A. THE CONTINUITY EQUATION

Highly ionized channels which develop at the final stages of a glow discharge just prior to arc breakdown are called streamers. The transition to arcing is then often due to streamer-breakdown. It should be noted here that the type of streamers to be analyzed are the non-self-sustaining discharge streamers that precede the arc, rather than the electron avalanche (onset streamers) type that occur earlier in the glow discharge phenomena.

Streamers are seen to issue from either electrode and appear always to precede the breakdown. These streamers propagate virtually instantaneously for our purposes as an ultrafast ionization wave. They consist of a partially ionized plasma column for which the conductivity is dependent directly on the charged particle density within the streamer [Ref. 4]. It will be shown that the collisions made by the charged particles within these streamers are the main source of heating of the surrounding gas. The value of this charged particle density may be determined by the application of the following continuity equation to the breakdown process.

$$\frac{\partial n}{\partial t} = \frac{D_a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) - U\frac{\partial n}{\partial r} - \alpha_2 n^2 - \alpha_3 n^3 \qquad (2.1)$$

Equation (2.1) gives the time rate of change of the charged particle density, n, as a function of radial position, r, and time, t, for the decaying streamer. It is written in cylindrical coordinates due to the natural form of the streamer. A schematic drawing of the streamer geometry is depicted in Figure 2.1. The right hand side (RHS) of Equation (2.1) consists of three classes of terms. The first term represents the effect of diffusion due to the high density gradient within the plasma channel. D_a is the ambipolar diffusion coefficient. This ambipolar diffusion results in the electrons and ions diffusing out of the original channel at essentially the same rate. Briefly, the lighter electrons having higher thermal velocities leave the plasma ahead of the slower, heavier ions. This produces a local positive space charge which retards the loss of electrons and accelerates the loss of the ions. The net effect is that the ambipolar diffusion coefficient is approximately twice the diffusion coefficient of the ions alone [Ref. 4].

Next consider terms 3 and 4 of the RHS of Equation (2.1) which represent two and three-body recombination respectively. Recombination is a de-ionization process which is then responsible here for the reduction in conductivity of the partially ionized gas. These processes occur when an electron and an ion collide at relatively low velocity and recombine to form a neutral atom. In order to conserve linear momentum a third body must be present. In two-body recombination

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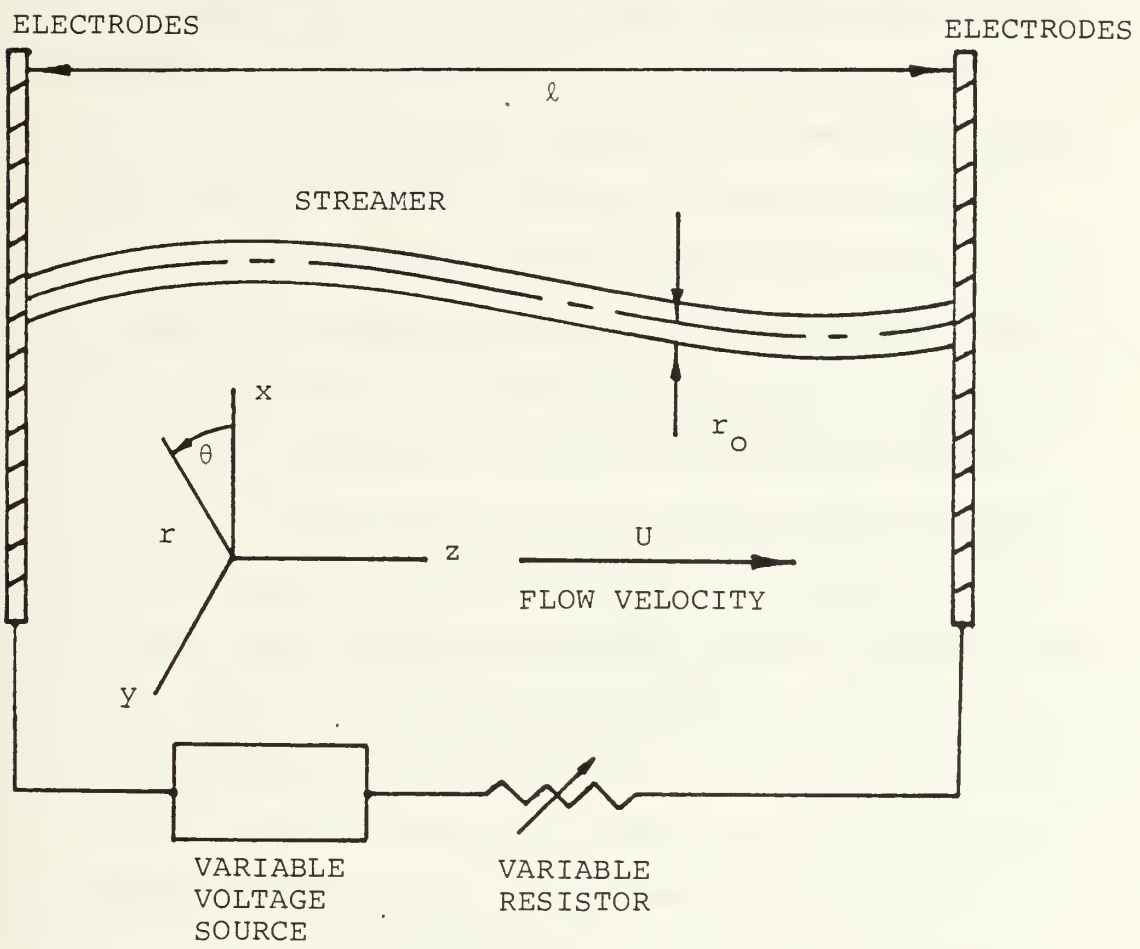

Figure 2.1. Streamer Geometry

this third body is a photon and, therefore, this process is sometimes called radiative recombination. For three-body recombination the third body is usually an ion or neutral atom [Ref. 4]. Due to the large complexity of electron loss mechanisms accurate values for the recombination coefficients are difficult to obtain. The values used in this thesis will be discussed in the next subsection.

The second term on the RHS is the convection term. This term is neglected since the characteristic time of convection (length/flow velocity) is much less than the characteristic recombination times associated with terms 3 and 4 [Ref. 2].

While the convection term in Equation (2.1) can be neglected, the diffusion term cannot since the diffusion coefficient D_a can increase by about three orders of magnitude between laminar and turbulent flow [Ref. 2]. This will cause the characteristic diffusion time to be on the same order of magnitude as the recombination time and some relationship will exist between diffusion and recombination which will affect the charged particle density distribution, n, in space and time.

Once Equation (2.1) is solved for n(r,t) the resulting distribution profile can be integrated to determine a conductance of the decaying streamer as a function of the streamer radius and time. This can be accomplished under varying turbulence level conditions by changing the effective value of the diffusion coefficient.

Equation (2.1) describes the lightning-discharge type phenomenon in a non-ionized medium. A more interesting situation is the case of the streamer as it appears in an ionized-sustainer discharge for the pumping of a gas laser [Ref. 5]. For this case, Equation (2.1) may be written as,

$$\frac{\partial n}{\partial t} - \frac{D_a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = \nu_i n - \alpha_2 n^2 - \alpha_3 n^3 \qquad (2.2)$$

where now a new first term is added to the RHS which represents a production of electrons and ions. The ionization coefficient, ν_i, is a function of the sustainer E/N and can be determined from Figure 2.2. The electric field referred to here is an existing axial electric field, that is, the "pumping field" that maintains the glow. Equations (2.1) and (2.2) are non-linear partial differential equations which will be solved numerically by the use of a finite difference approximation scheme.

B. THE ENERGY EQUATION

In addition to the continuity equation, a suitable energy equation must be considered in order to describe the temperature distribution introduced into the gas by the glow discharge streamer. For thermal stability to exist, a balance must be created such that the time rate of change of the internal energy is zero because the loss of heat through thermal conductivity and the addition of energy from Ohmic

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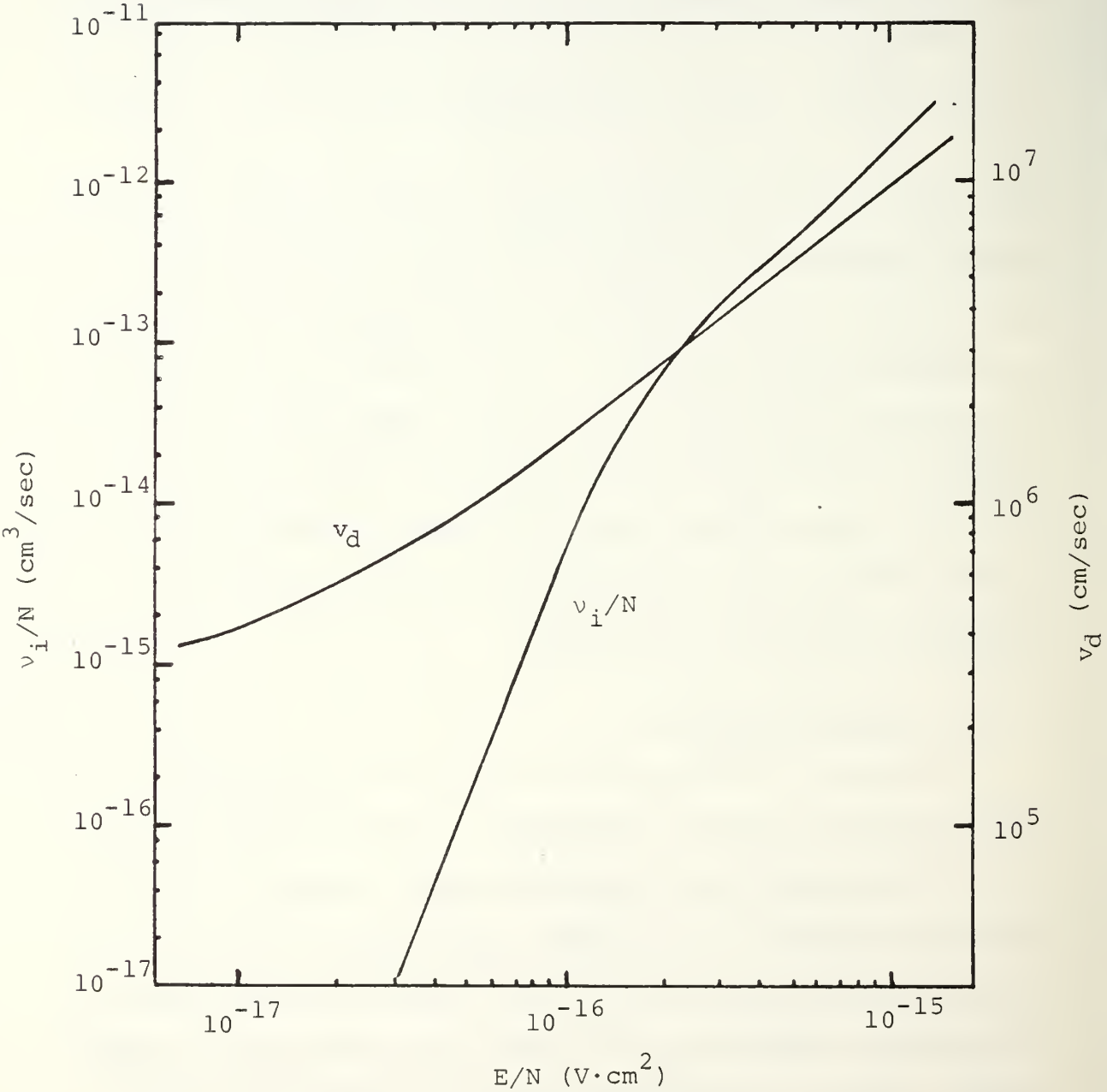

Figure 2.2. Relevant Properties of Nitrogen

heating must balance each other. The following equation represents this relationship.

$$\rho \frac{De}{Dt} = \vec{E} \cdot \vec{j} - \nabla \cdot \vec{q} \qquad (2.3)$$

The first term on the RHS of Equation (2.3) represents Joule or Ohmic heating where, $\vec{E}$, is the electric field strength and, $\vec{j}$, is the current density. The second term is the heat flux loss due to thermal conductivity and radiation. The total derivative,

$$\frac{De}{Dt} = \frac{\partial e}{\partial t} + v_r \frac{\partial e}{\partial r} + v_\theta \frac{\partial e}{\partial \theta} + v_z \frac{\partial e}{\partial z} \qquad (2.4)$$

reduces to, $\frac{\partial e}{\partial t}$, since the model under consideration here allows for conduction only in the "z" direction. It is further assumed that the change of the internal energy along the length of the streamer is small, the radiation emitted primarily perpendicular to the streamer centerline has a negligible effect and, due to the short characteristic times involved, the heating occurs at an essentially constant volume. With these assumptions in mind, Equation (2.3) becomes,

$$\rho C_v \frac{\partial T}{\partial t} - K\nabla^2 T = \sigma E_z^2 \qquad (2.5)$$

where, ρ, is the gas density, σ, is the electrical conductivity of the plasma (recalling that $j = \sigma E_z$) and K is the

thermal conductivity. Equation (2.5) gives the temperature, T, as a function of the radius and time.

The conductivity, σ, is given by,

$$\sigma = e n_e \mu_e \tag{2.6}$$

where, e, is the elementary electronic charge in coulombs, n_e, is the number of electrons and, μ_e, is the electron mobility. The mobility is also a function of E/N and is defined as the ratio of the electron drift velocity, v_d, to the applied external electric field. The value of v_d is given in Figure 2.2 as a function of E/N.

It will be shown later that for the σE_z^2 term from Equation (2.5), E_z is essentially constant and since σ is directly proportional to n the solution of Equation (2.2) represents the single source of heating that drives the temperature distribution obtained from Equation (2.5). The consequence of this is that a mathematical model describing the temporal and spatial characteristics of the streamer can now be developed. Furthermore, the effects of turbulence as a stabilizing medium can then be studied since the thermal conductivity, K, and the ambipolar diffusion coefficient, D_a, are the only parameters affected by turbulence in Equations (2.1), (2.2) and (2.5).

C. SOLVING THE EQUATIONS

An effective means of generalizing a problem such as this is to non-dimensionalize the equations. To begin then,

after some algebraic manipulation, Equation (2.1) can be written in the form,

$$\frac{1}{\alpha_2 n_o}\frac{\partial \hat{n}}{\partial t} - \left[\frac{D_a}{n_o \alpha_2 r_o^2}\right]\frac{\partial}{\partial \hat{r}}\left(\hat{r}\,\frac{\partial \hat{n}}{\partial \hat{r}}\right) = -\hat{n}^2 - \frac{\alpha_3 n_o}{\alpha_2}\hat{n}^3 \qquad (2.7)$$

where $\hat{n} = n/n_o$ and $\hat{r} = r/r_o$. "n_o" and "r_o" are the initial values for the charged particle density and the streamer radius. The following additional definitions will prove useful in this model,

$$\beta \equiv \frac{\alpha_2 n_o r_o^2}{D_a} \qquad (2.8)$$

$$\alpha \equiv \frac{\alpha_3 n_o}{\alpha_2} \qquad (2.9)$$

$$\hat{t} \equiv \frac{t}{\tau_R} \qquad (2.10)$$

The parameter, β, is an inverse measure of the turbulence level since the ambipolar diffusion coefficient is smaller in laminar flow (large β) and larger in a turbulent flow (small β). If one neglects the diffusion term in Equation (2.1), then for two-body recombination, direct integration yields,

$$n = \frac{n_o}{1 + \alpha_2 n_o t} \qquad (2.11)$$

This ion loss formula [Ref. 2] then shows that the characteristic recombination time, τ_R, is simply $1/\alpha_2 n_o$.

Incorporating the above definitions allows Equation (2.7) to be written in the non-dimensionalized form,

$$\frac{\partial \hat{n}}{\partial \hat{t}} - \frac{1}{\beta}\frac{\partial}{\partial \hat{r}}\left(\hat{r}\,\frac{\partial \hat{n}}{\partial \hat{r}}\right) = -\hat{n}^2 - \alpha\hat{n}^3 \tag{2.12}$$

which lends itself to a numerical solution by substituting the appropriate finite difference approximation formula [Ref. 6] for the differentials. After some manipulation, Equation (2.12) can be approximated by,

$$\hat{n}_{i,j+1} = \{\Delta\hat{t}[\frac{1}{\beta}(A+B) - C]\} + \hat{n}_{i,j} \tag{2.13}$$

where,

$$A = \frac{1}{\Delta\hat{r}^2}(\hat{n}_{i+1,j} - 2\hat{n}_{i,j} + \hat{n}_{i-1,j}) \tag{2.14}$$

$$B = \frac{1}{2\hat{r}\Delta\hat{r}}(\hat{n}_{i+1,j} - \hat{n}_{i-1,j}) \tag{2.15}$$

$$C = (\hat{n}^2_{i,j} + \alpha\hat{n}^3_{i,j}) \tag{2.16}$$

In this scheme, 'i' represents position perpendicular to the streamer centerline and 'j' represents relative position in time after some initial condition. Equations (2.13) through (2.16) form the basic algorithm used by the Fortran program discussed in Appendix A.

The accuracy of the charged particle density obtained by this type of differencing scheme depends on the step sizes $\Delta\hat{t}$ and $\Delta\hat{r}$. The steps must be smaller than the smallest characteristic time of the phenomena described. Therefore, $\Delta\hat{t}$ is taken as 0.1 or in real time $\tau_R/10$ [Ref. 6]. Primarily for programming convenience, $\Delta\hat{r}$ is also taken as 0.1. Additionally, the value of β is limited by the following stability criterion [Ref. 2].

$$\beta > \frac{2\Delta\hat{t}}{\Delta\hat{r}^2} \tag{2.17}$$

Returning to Equation (2.2), it is seen that the addition of the ionization term to the RHS requires a modification to Equation (2.16) which now becomes,

$$C_1 = [\hat{n}^2_{i,j} + \alpha\hat{n}^3_{i,j} - \hat{n}_{i,j}] \tag{2.18}$$

Equation (2.5) can be non-dimensionalized in a similar manner by writing it in the form,

$$\frac{T_o}{\tau_A}\frac{\partial\hat{T}}{\partial\hat{t}_A} - \left[\frac{KT_o}{\rho C_v r_o^2}\right]\frac{1}{\hat{r}}\frac{\partial}{\partial\hat{r}}\left[\hat{r}\frac{\partial\hat{T}}{\partial\hat{r}}\right] = \frac{\sigma E_z^2}{\rho C_v} \tag{2.19}$$

where τ_A is defined as,

$$\tau_A \equiv \frac{T_o \rho C_v}{\sigma_o E_{z_o}^2} \tag{2.20}$$

A check of Equation (2.20) reveals that τ_A has the units of seconds and, therefore, is a characteristic time for the energy equation. If one now defines the following additional relations,

$$\hat{t}_A \equiv \frac{t}{\tau_A} \tag{2.21}$$

$$\hat{T} \equiv \frac{T}{T_o} \tag{2.22}$$

$$\beta_A \equiv \frac{\sigma E_{z_o}^2 r_o^2}{K T_o} \tag{2.23}$$

then, it can be shown that, Equation (2.19) becomes the following non-dimensional relation,

$$\frac{\partial \hat{T}}{\partial \hat{t}_A} - \frac{1}{\beta_A \hat{r}} \frac{\partial}{\partial \hat{r}}\left(\hat{r} \frac{\partial \hat{T}}{\partial \hat{r}}\right) = \frac{\sigma}{\sigma_o}\left(\frac{E_z^2}{E_{z_o}^2}\right) \tag{2.24}$$

where, T_o, is the initial temperature, which for this work, is taken as 300° Kelvin. It should be clear here that, β_A, is now a non-dimensional parameter whose value depends only on the thermal conductivity, K. This parameter then serves the same function for the energy equation as does β for the continuity equation, that is, it allows the influence of turbulence to be introduced into the model. The value of the electric field strength, E_z, from the RHS of Equation (2.24) can be estimated by assuming that the initial value of the externally applied field, E_o, is given by [Ref. 8],

$$E_o = E + \frac{I\Omega}{\ell} \tag{2.25}$$

A reasonable value for Ω/ℓ might be on the order of 10,000 Ω/m, for which Equation (2.25) becomes,

$$E_o = E[1 + \frac{I}{E}(10,000)] \tag{2.26}$$

where I/E for the streamer can be shown to be,

$$\frac{I}{E} = \frac{e(\mu_e N)}{N} \int_0^r 2\pi r n_e \, dr \tag{2.27}$$

and,

$$N = \frac{P}{kT} \tag{2.28}$$

"N" is the total number of particles present in the gas at a pressure, P, and a temperature, T. The Boltzman constant is k = 1.38E-23 J/°K. The integral in Equation (2.27) is a measure of the conductance of the streamer and is evaluated in Chapter III by integrating the charged particle density distribution obtained from Equation (2.1). The initial value of this integral is simply $\pi n_o r_o^2$. Using estimates of n_o and r_o to be 3.125E18 m^{-3} and 4.0E-4 m respectively [Ref. 7], Equation (2.26) can be solved for E_z becoming,

$$E_{z_o} = \frac{E_o}{1 + 0.0006} \simeq E_o \qquad (2.29)$$

where the value of $\mu_e N$ in Equation (2.27) can be obtained from reference tables to be 5.257E24 $(v\text{-}m\text{-}s)^{-1}$ for nitrogen. Results to be discussed in Chapter III show that the value of the integral drops rapidly to a small and nearly steady value (on the order of 0.01) such that the denominator of Equation (2.29) approaches unity in very short times.

The results of the previous paragraph then show that the RHS of Equation (2.24) is simply $\hat{n}$ since σ/σ_o is $(en_e\mu_e/en_o\mu_e)$ and $E_z^2/E_{z_o}^2$ is approximately one. So now finally, Equation (2.24) can be written as,

$$\frac{\partial \hat{T}}{\partial \hat{t}_A} - \frac{1}{\beta_A \hat{r}} \frac{\partial}{\partial \hat{r}}\left[\hat{r} \frac{\partial \hat{T}}{\partial \hat{r}}\right] = \hat{n} \qquad (2.30)$$

Equation (2.30) then shows that the temperature distribution due to the presence of the streamer is affected only by the charged particle density.

As for the continuity equations, Equation (2.30) can be solved by introducing the appropriate finite difference formula as follows (after some rearranging of terms),

$$\hat{T}_{i,j+1} = \Delta\hat{T}\left[\frac{1}{\beta_A}(A+B) + n_{i,j}\right] + \hat{T}_{i,j} \qquad (2.31)$$

where,

$$A = \frac{1}{\Delta\hat{r}^2}[\hat{T}_{i+1,j} - 2\hat{T}_{i,j} + \hat{T}_{i-1,j}] \qquad (2.32)$$

and,

$$B = \frac{1}{2\hat{r}\Delta\hat{r}}[\hat{T}_{i+1,j} - \hat{T}_{i-1,j}] \qquad (2.33)$$

The Fortran program discussed in Appendix A solves Equation (2.31) simultaneously with Equation (2.1) or (2.2) depending on the case being considered by using the new value of $\hat{n}$ calculated from each step as an input to the calculation of the new temperature value. The time step for the temperature calculation is adjusted such that the real time step for the two equations is equal.

D. MODELING THE STREAMER

The results of Chapter III show that appropriate selection of the controlling parameters for Equations (2.2) and (2.30) will produce a stable system in space and time. Then the question is: what effect does the momentary appearance of a streamer have on this stable situation? If a local perturbation of the electric field occurs, then the ionization coefficient will momentarily be raised which will in turn raise the charged particle density in the region of the perturbation. Figure 2.2 provides the data for estimating the magnitude of this perturbation by noting the relative increase in E/N for a corresponding increase in ν_i/N. The

ratio of these values can then be used to determine a simple multiplier (labelled Z and Z_A) for the $\hat{n}$ terms appearing on the RHS of Equations (2.2) and (2.30) to simulate the effect of the streamer. Therefore, one can write,

$$\frac{\partial \hat{n}}{\partial \hat{t}} - \frac{1}{\beta_A \hat{r}} \frac{\partial}{\partial \hat{r}} \left(\hat{r} \frac{\partial \hat{n}}{\partial \hat{r}} \right) = Z\hat{n} - \hat{n}^2 - \alpha \hat{n}^3 \qquad (2.34)$$

and

$$\frac{\partial \hat{T}}{\partial \hat{t}_A} - \frac{1}{\beta_A \hat{r}} \left(\hat{r} \frac{\partial \hat{T}}{\partial \hat{r}} \right) = Z_A \hat{n} \qquad (2.35)$$

where Z and Z_A are the multipliers discussed. For the finite difference approximations, these two additions are easily handled for Equations (2.2) and (2.30) respectively, as follows.

$$C_2 = [\hat{n}^2_{i,j} - \alpha \hat{n}^3_{i,j} - Z_i \hat{n}_{i,j}] \qquad (2.36)$$

$$\hat{T}_{i,j+1} = \Delta \hat{t} \left[\frac{1}{\beta_A} (A+B) + Z_A \hat{n}_{i,j} \right] + \hat{T}_{i,j} \qquad (2.37)$$

The Fortran program incorporates this concept by utilizing a separate subroutine called "STREMR" which is identical to the subroutine "DENSTY" except for the addition of Equations (2.36) and (2.37). In this scheme then, the streamer is modeled by calling up this subroutine when desired and then observing the reaction of the system over time.

III. RESULTS AND DISCUSSION

A. NON-IONIZED ENVIRONMENT CASE

In this section the effects of turbulence on the streamer in a non-ionized environment are considered. Equation (2.12) applies to this situation for which Figure 3.1 shows a typical plot of the charged particle density versus radial position perpendicular to the centerline in a fully laminar flow. Note that the abscissa and the ordinate are expressed in the non-dimensional coordinates of $\hat{r} = r/r_o$ and $\hat{n} = n/n_o$, respectively. There are five curves plotted which show the charged particle density distribution profile from the initial conditions at time zero in steps as specified in the legend to the profile at 3000 recombination times (TR). "TR" is the computer's version of "τ_R" and will be used here in the text. Now, for this plot, and except for Figures 3.11 and 3.12, for all the figures in this section, n_o = 3.125E18 m^{-3} and α_2 = 2.0E-12 m^3/sec [Ref. 7]. These values then equate to a recombination time of 0.16 microsecond, and so, 1000TR is equivalent to 0.16 millisecond in real time.

In the laminar case depicted in Figure 3.1 it is clearly seen that the centerline charged particle density drops off in time in accordance with Equation (2.11) which in non-dimensional form would be,

$$\hat{n}_{\mathfrak{C}} = \frac{1}{1 + \hat{t}} \qquad (3.1)$$

REPRODUCED AT GOVERNMENT EXPEN

CHARGED PARTICLE DENSITY PROFILE

IN LAMINAR FLOW

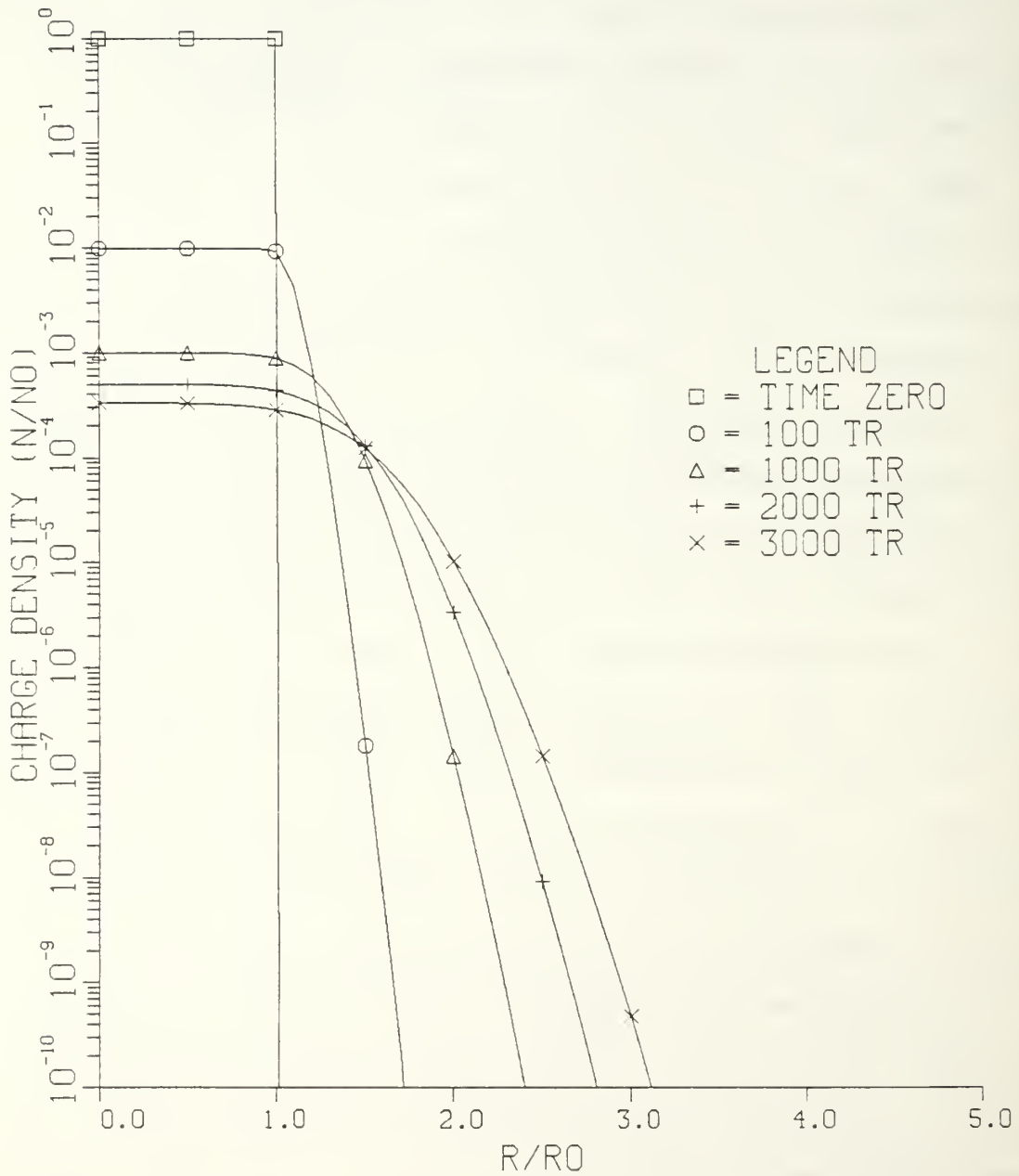

FIGURE 3.1

For example, at 1000TR, $\hat{n}_{\mathcal{L}}$ = 1.0E-3. This decay is due primarily to recombination only since, in this case, beta = 50,000, and therefore, the diffusion term becomes very small. For longer times, diffusion comes into play and some spreading of the charge profile is seen.

Figure 3.2 shows the effect of introducing increasing amounts of turbulence into the flow. This plot compares the different charged particle profiles as they would appear at 0.32 msec for beta values of 50 (fully turbulent), 500, 5000 and 50,000 (fully laminar). Note that the 2000TR curve from Figure 3.1 is identical to the beta = 50,000 curve in Figure 3.2. The effect of the turbulence is clearly seen to spread out the profile and to lower $\hat{n}$ on the centerline as well [Ref. 2].

As was mentioned previously, the conductance of the decaying streamer can be determined by integrating the charged particle density over the radius. This can be derived by "inverting" Ohm's law and introducing the conductivity expression for a partially ionized plasma for which the current can now be expressed as [Ref. 4],

$$I = GV = (\frac{\sigma A}{\ell})V = (\frac{e^2 n_e A}{m_e \nu_e \ell})V \qquad (3.2)$$

where G is the conductance, or the reciprocal of the resistance, in the familiar relation, V = IR. 'σ' is the conductivity, 'A' the cross-sectional area and 'ℓ' is the

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CHARGED PARTICLE DENSITY PROFILE

IN TURBULENT FLOW

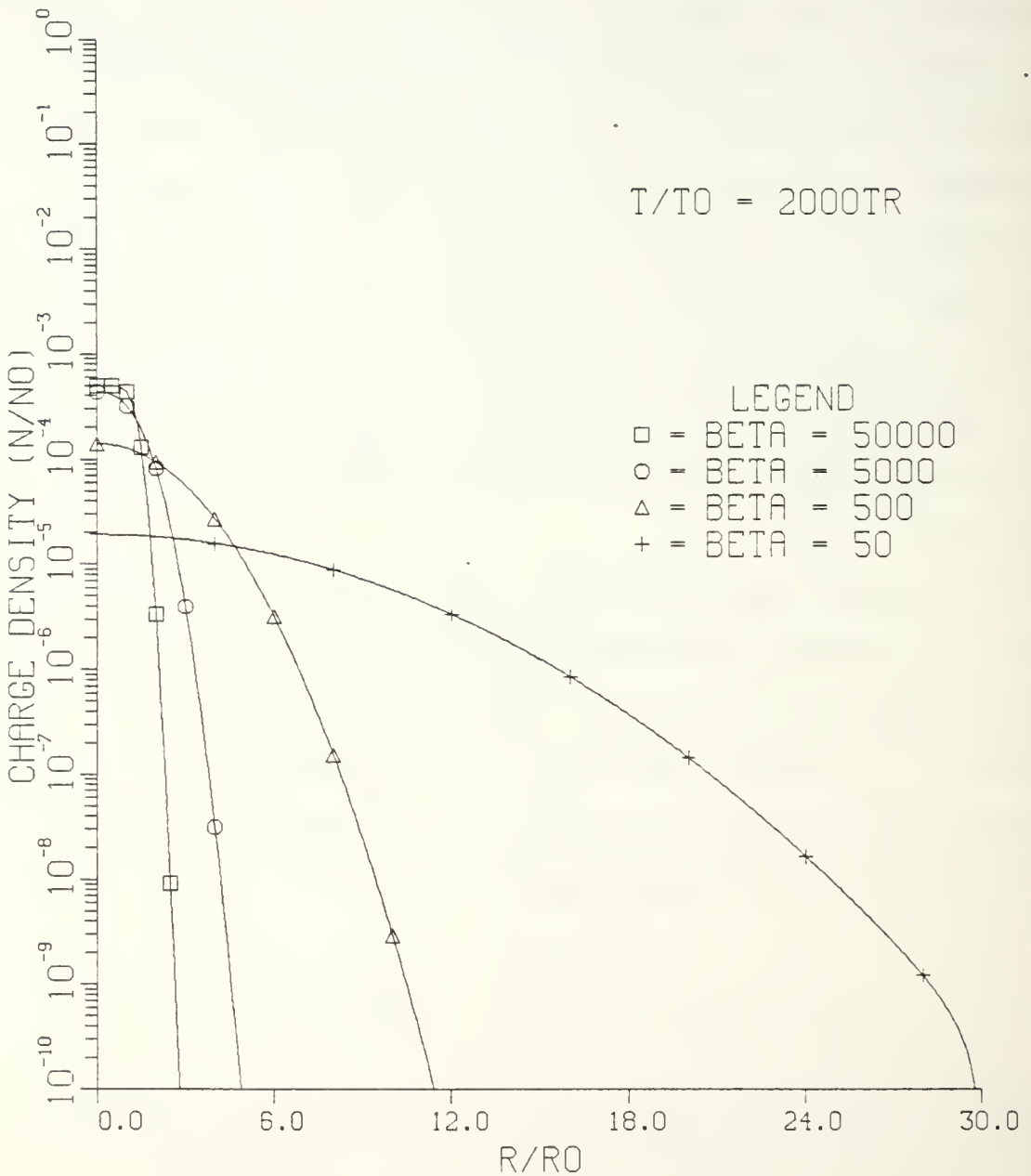

FIGURE 3.2

length of the streamer (or the distance between the electrodes in Figure 2.1 for this case). Additionally, m_e and ν_e are the electron mass and the electron collision frequency, respectively.

For the decaying streamer, the radius, and therefore the area, is changing but the length is constant. Since all the other factors within the brackets of Equation (3.2) are also constant, 'G' can then be expressed as,

$$G = \frac{e^2}{\ell m_e \nu_e} \int_0^{\hat{r}} \hat{n}\, 2\pi\, \hat{r}\, d\hat{r} \qquad (3.3)$$

where gamma is now defined as the value of the integral in the above equation. Gamma has the units of reciprocal meters, and is determined in the program, "CHARGE" (Appendix A), by the subroutine, "CNDUCT." Gamma is then directly proportional to the conductance.

Figure 3.3 shows gamma plotted over time out to 100TR. Note that gamma drops in value by 90% in only about 10TR (1.6 microsec). After this time the value levels off to about 0.04 at 100TR and then beyond the range of Figure 3.3 the value drops very slowly to about 0.001 at 10,000TR (1.6 msec).

Figure 3.3 is for fully laminar flow. Introducing turbulence into the flow causes a very slight increase in gamma over all time for the complete streamer. Figure 3.4

REPRODUCED AT GOVERNMENT EXPEN

GAMMA VS TIME

IN LAMINAR FLOW

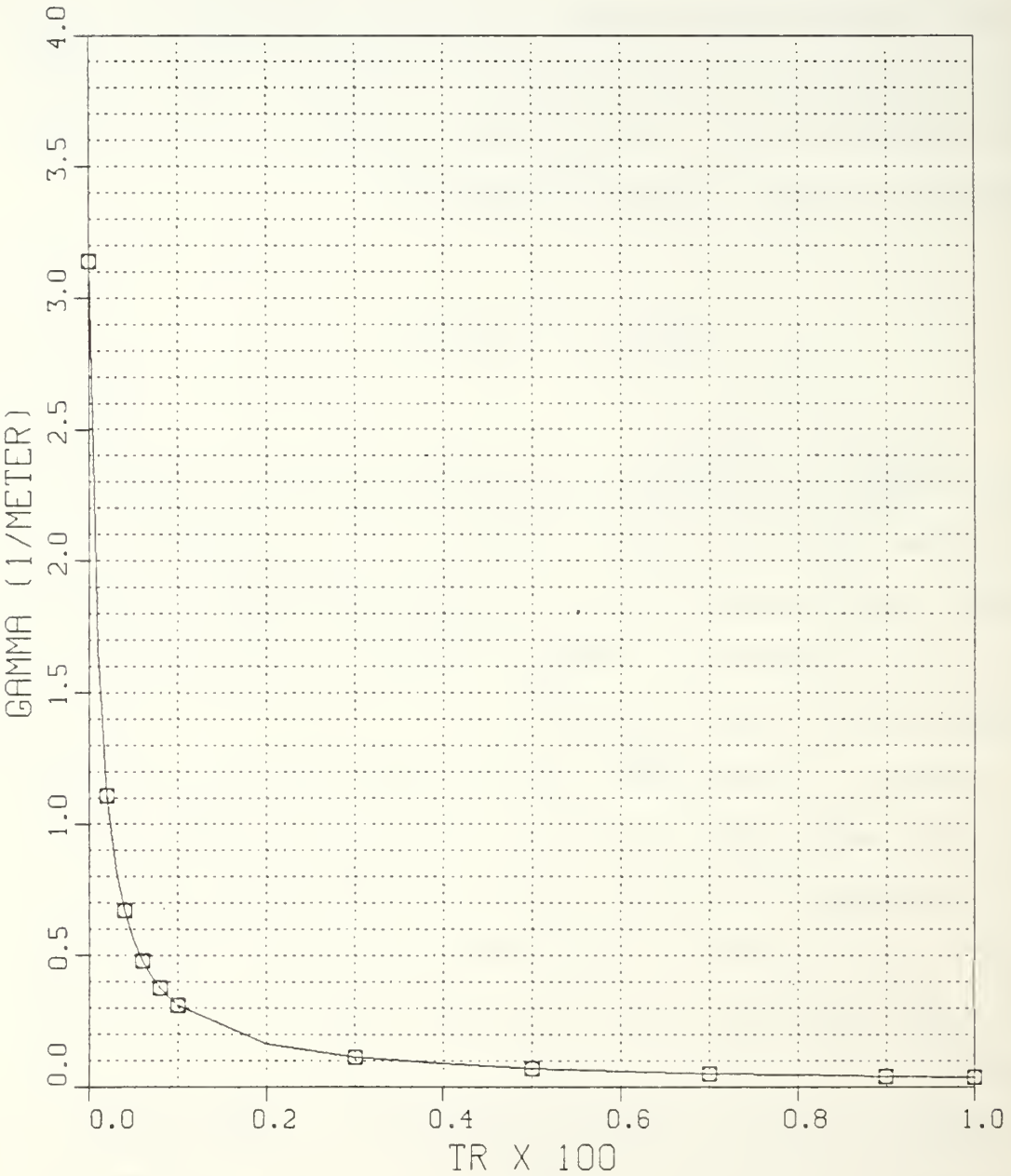

FIGURE 3.3

REPRODUCED AT GOVERNMENT EXPENSE

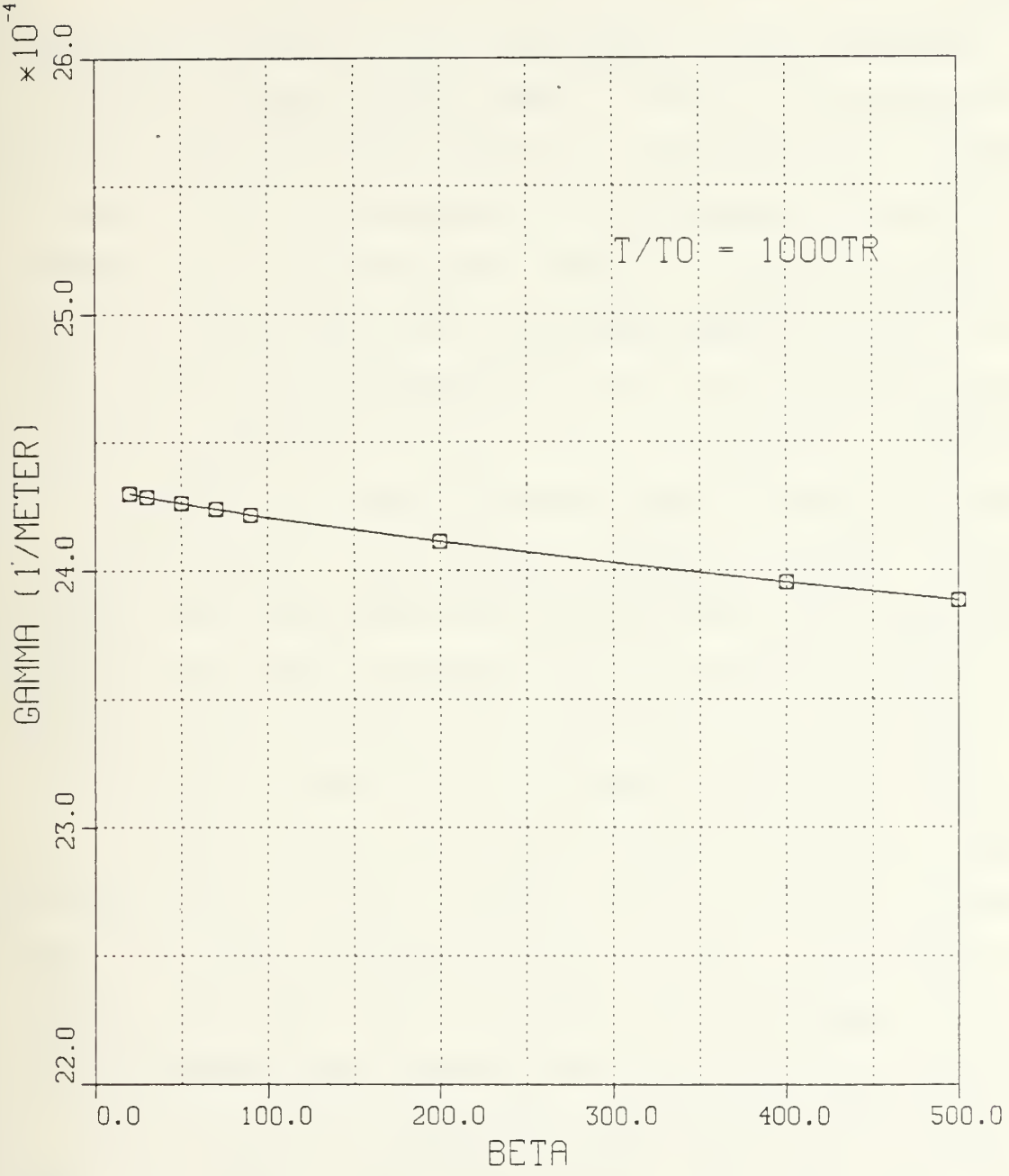

FIGURE 3.4

shows this effect at 1000TR. Figure 3.5 shows the value of gamma within the original streamer tube channel only. Here it is seen that for small beta (high turbulence) the conductance is lower than in the laminar flow condition.

Referring back to Figure 3.2, and with these conductance characteristics in mind, it seems rather apparent that one effect of the turbulence is to delay the possibility of a transition to arcing since the conductance is lower along the original channel. Another consideration is what happens to a follow-on streamer appearing after the charged particle density profile of the first streamer has died away for say, 2000TR as in Figure 3.2. In a laminar flow there would be a distinct path of least resistance along the original channel. However, in the turbulent flow, the profile is spread out over a much larger radius and then, this "follow-on streamer" essentially has no favored "route" for arcing.

Equation (2.30) showed that the charged particle density was the sole source of heating in this model. Figure 3.6 shows the result of solving Equation (2.30) in a fully laminar flow for an E/N of 1.635E-17 V-cm^2, at one atmosphere, and 300°K for an electric field strength of 4.0E4 V/m. Note that the temperature rise amounts to less than about 0.05% in 1000TR and then rises very slightly over time eventually peaking out at about 4000TR and then falling back slowly. This would seem to be a logical behavior for this case since the charged particle density that is heating the gas is also

REPRODUCED AT GOVERNMENT EXPENSE

GAMMA VS BETA

FOR STREAMER TUBE

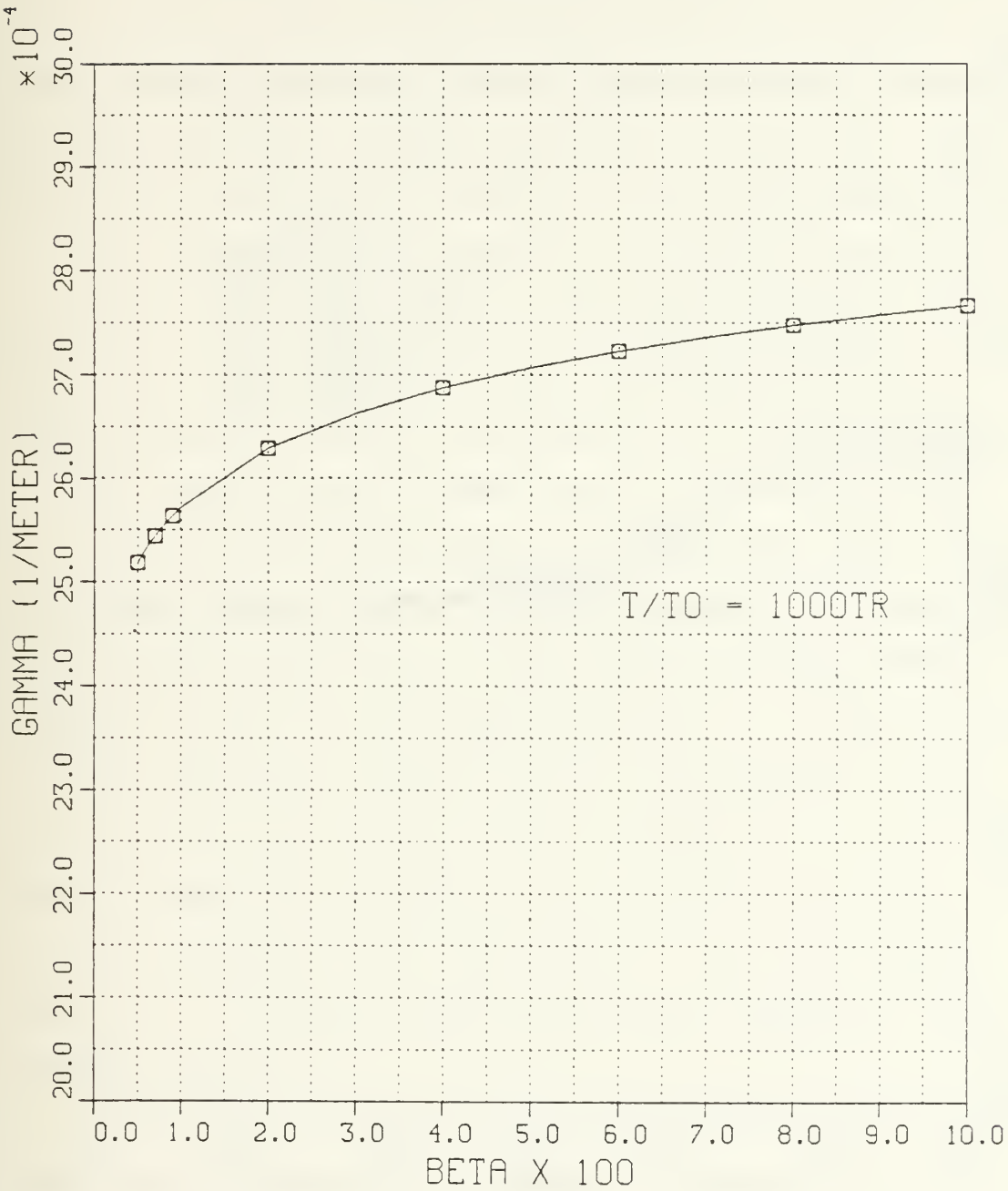

FIGURE 3.5

REPRODUCED AT GOVERNMENT EXPENS

TEMPERATURE PROFILE

FOR E = 4.0E4 V/M

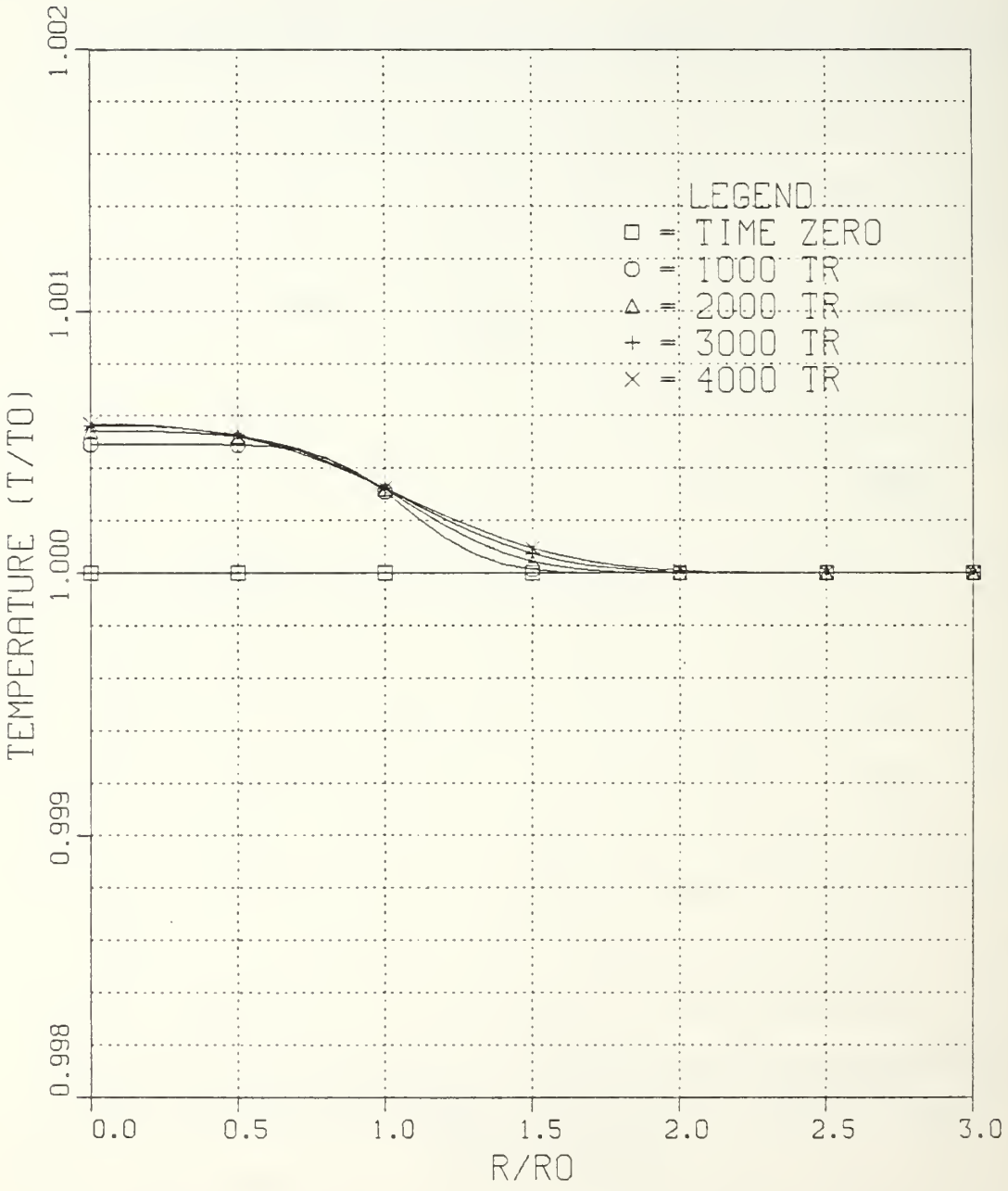

FIGURE 3.6

dying away. The question is then; what happens if one increases the electric field?

Figures 3.7 through 3.10 show the effects of increasing E_o from 1.0E6 V/m (a typical value of interest for lasers) to 8.0E6 V/m. Note that β_A changes for each of these cases. This is due to the fact that the electron drift velocity is a function of E/N, which is plotted in Figure 2.2. Since the drift velocity changes, the electron mobility changes which, in turn, produces a new conductivity value for each different electric field strength case. Recalling Equations (2.20) and (2.23), one can see that each case has a different characteristic time and a different β_A. For increasing E_o, τ_A gets smaller while β_A gets larger.

The main difficulty with this is that the time step for the energy equation solution must be adjusted to ensure that the continuity equation is "feeding" the charged particle density values into the energy equation at the correct rate. For the continuity equation, the non-dimensionalized time step is 0.1 (τ_R/10 in real time). Therefore, the true time for one time step (one "DT" in the program) is simply $DT \times \tau_R$. Similarly, the true time for the energy equation is $DTA \times \tau_A$. It follows then that the value needed for DTA is simply $(DT \times \tau_R)/\tau_A$.

Returning to Figures 3.7 through 3.10, it can be clearly seen that increasing E_o increases the temperature rise, as would be expected. The effects of longer times is the same as the case for Figure 3.6 but requires much longer computing

REPRODUCED AT GOVERNMENT EXPEN

TEMPERATURE PROFILE

FOR E = 1.0E6 V/M

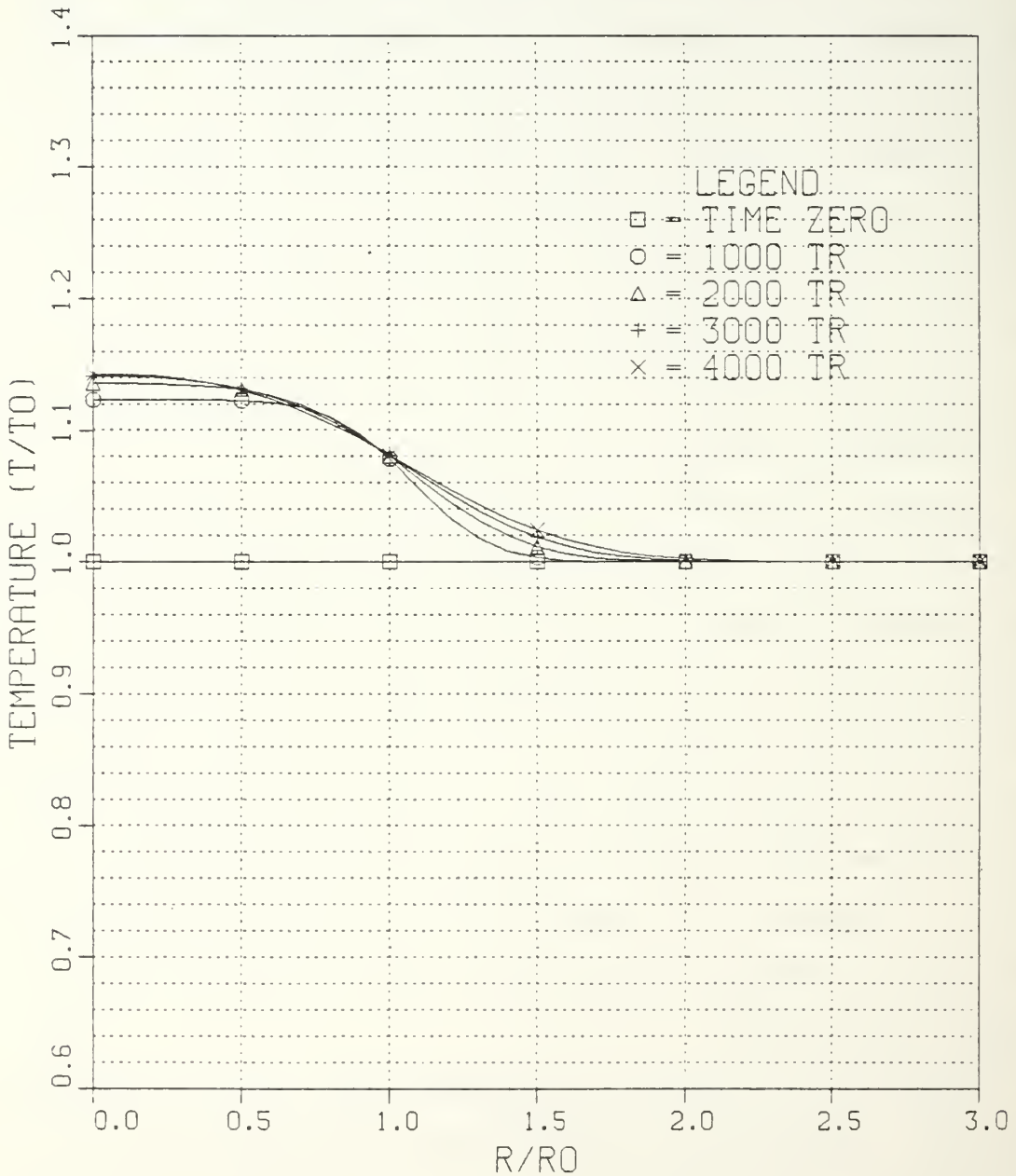

FIGURE 3.7

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

FOR E = 2.0E6 V/M

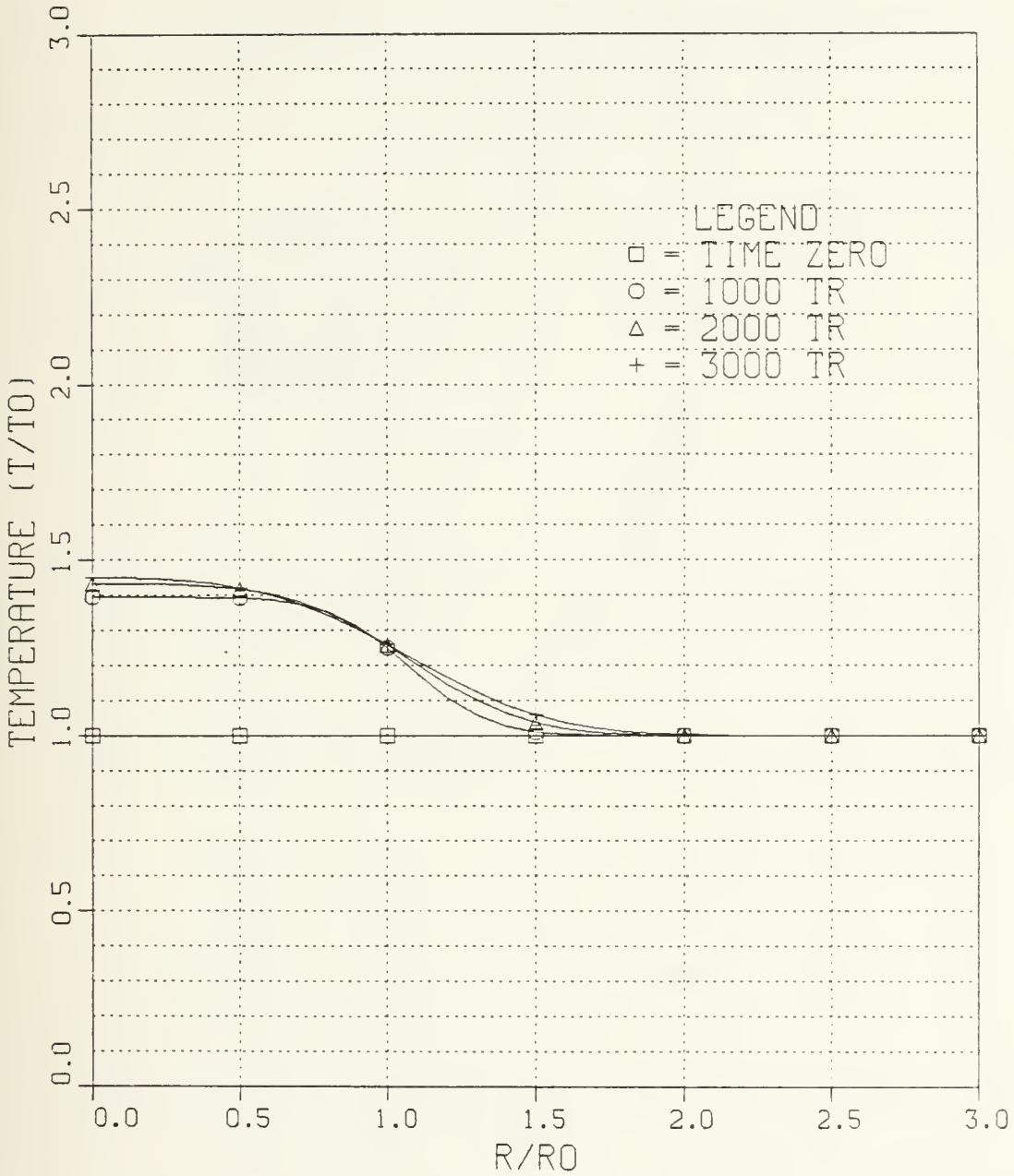

FIGURE 3.8

REPRODUCED AT GOVERNMENT EXPEN

TEMPERATURE PROFILE

FOR E = 4.0E6 V/M

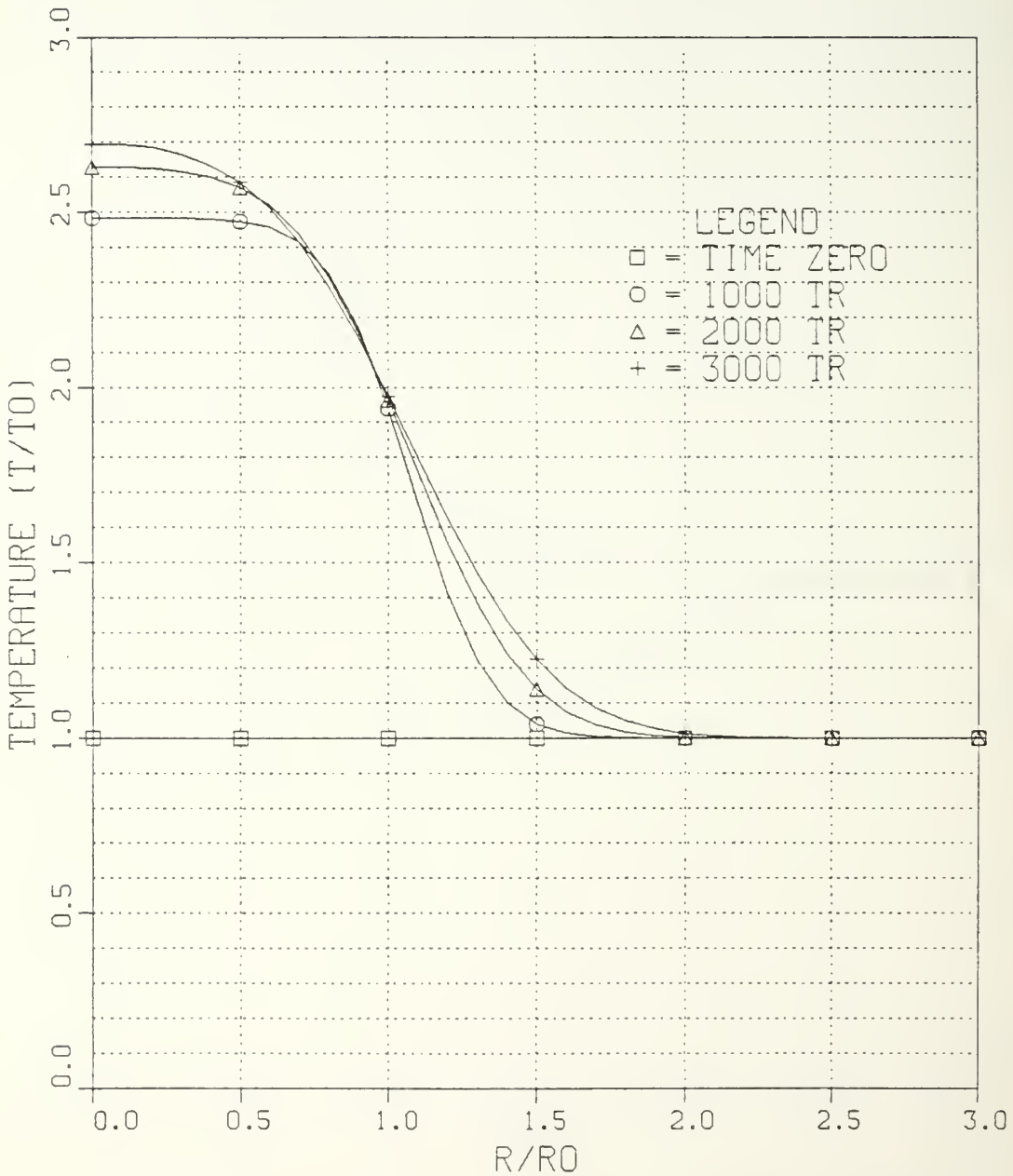

FIGURE 3.9

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

FOR E = 8.0E6 V/M

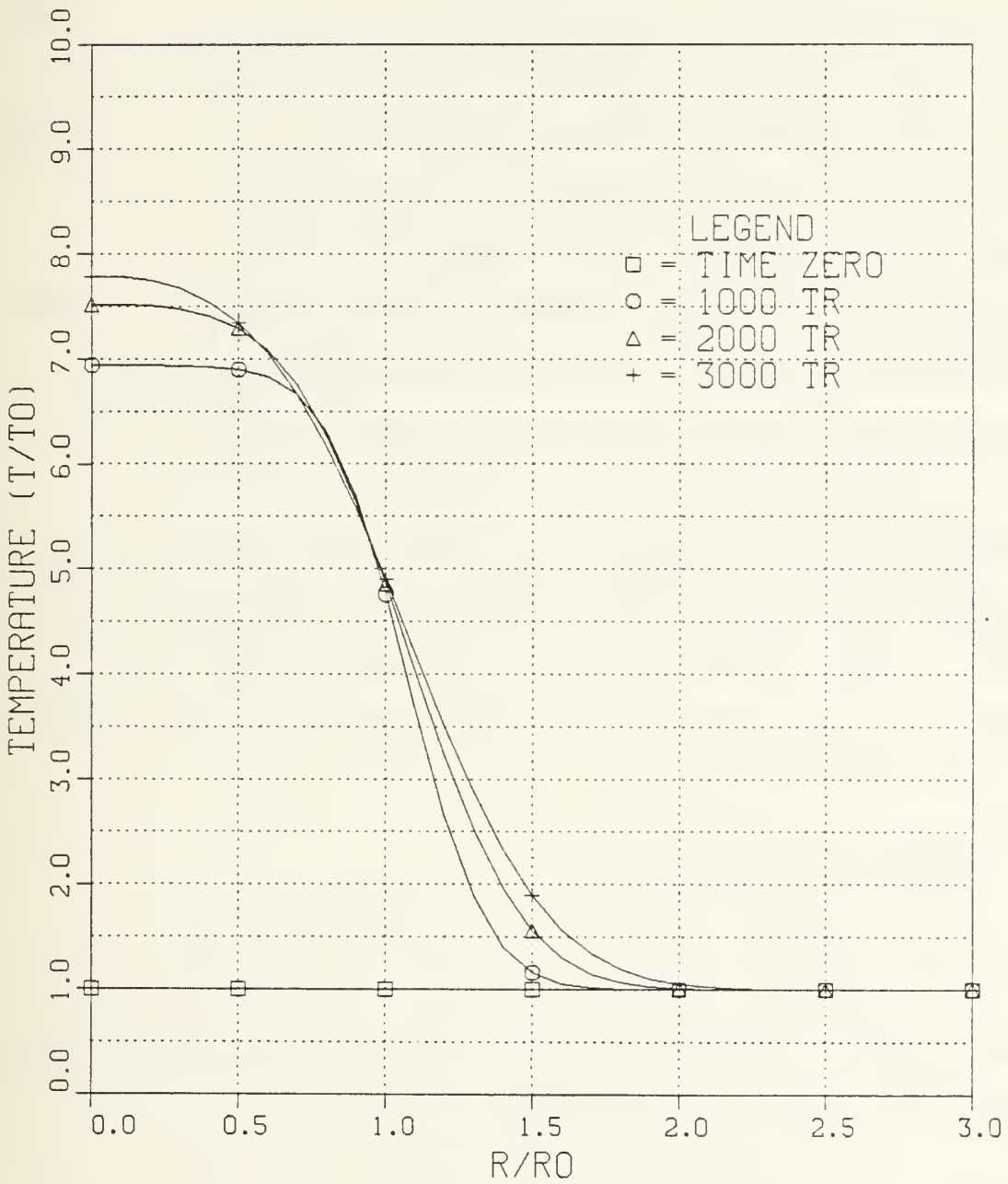

FIGURE 3.10

times to reach the peak and to begin falling back. The effect of turbulence on the cases of interest, say 4.0E4 V/m to 1.0E6 V/m, is to essentially eliminate the temperature rise.

One remaining question for this non-ionized case is what effect would increasing n_o have? Figure 3.11 shows that at E_o = 1.0E6 V/m and n_o increased by a factor of 10 to 3.125E19/m^3 produces a more squared off profile than that depicted in Figure 3.7 suggesting that the temperature dies rapidly outside of the immediate influence of the streamer. Figure 3.12 shows the effect of another 10-fold increase in N. Here the resulting temperature rise drops off at an even steeper rate at 2000TR than for Figures 3.7 and 3.11. Recalling that $\tau_R = 1/\alpha_2 n_o$, it is clear that increasing n_o decreases the characteristic recombination time significantly which, in turn, lowers $\hat{n}$ such that there is a noticeable reduction in the temperature profile outside of this immediate streamer influence.

B. IONIZED ENVIRONMENT CASE

In this section the effects of a glow discharge in an environment where a steady glow exists is considered. For this situation, Equation (3.4) is the applicable non-dimensional form of the governing continuity equation.

$$\frac{\partial \hat{n}}{\partial \hat{t}} - \frac{1}{\beta \hat{r}} \frac{\partial}{\partial \hat{r}}\left(\hat{r} \frac{\partial \hat{n}}{\partial \hat{r}}\right) = \hat{n} - \hat{n}^2 - \alpha \hat{n}^3 \qquad (3.4)$$

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

FOR NO = 3.125E19 M

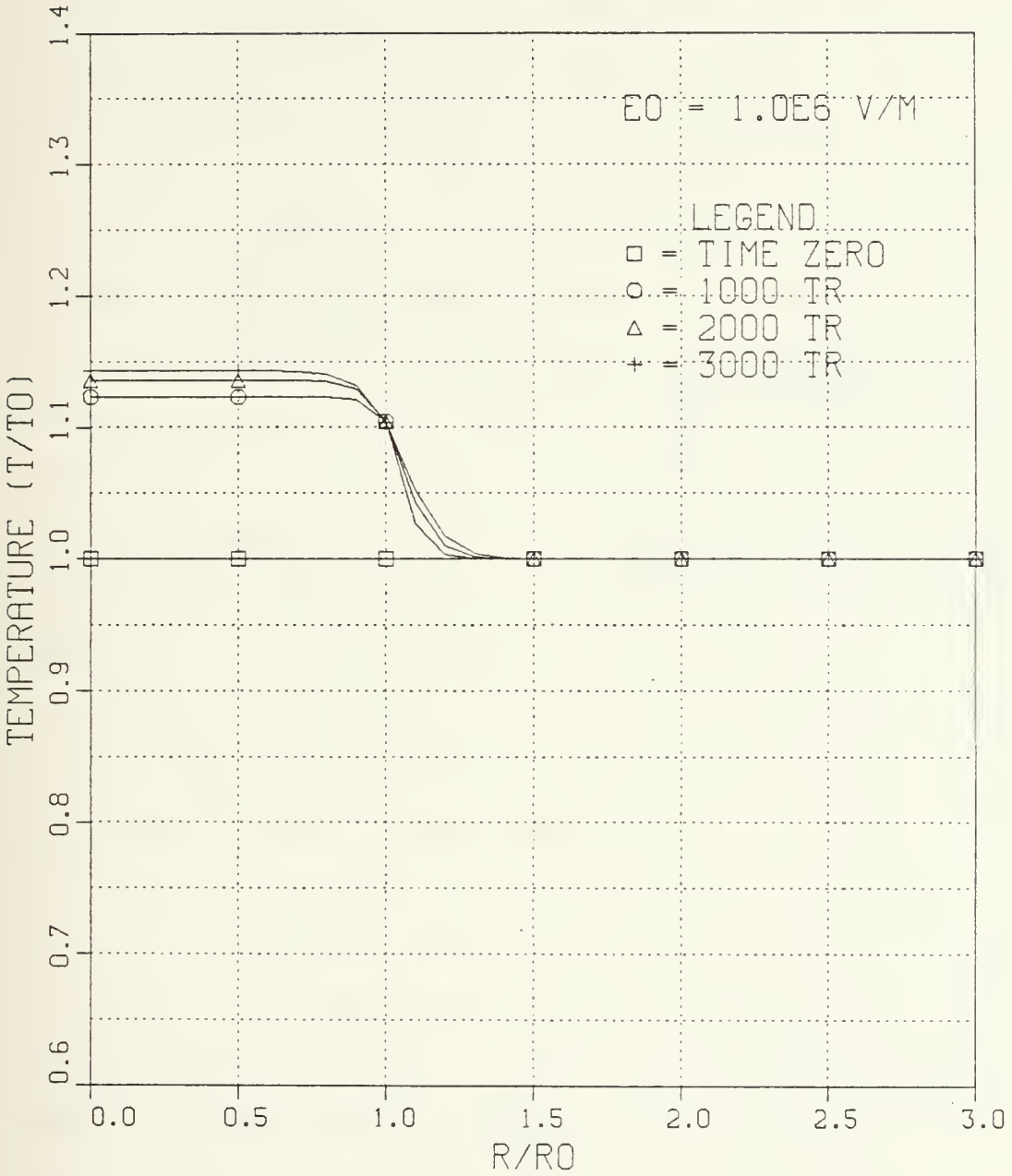

FIGURE 3.11

REPRODUCED AT GOVERNMENT EXPEN

TEMPERATURE PROFILE

FOR NO = 3.125E20 M

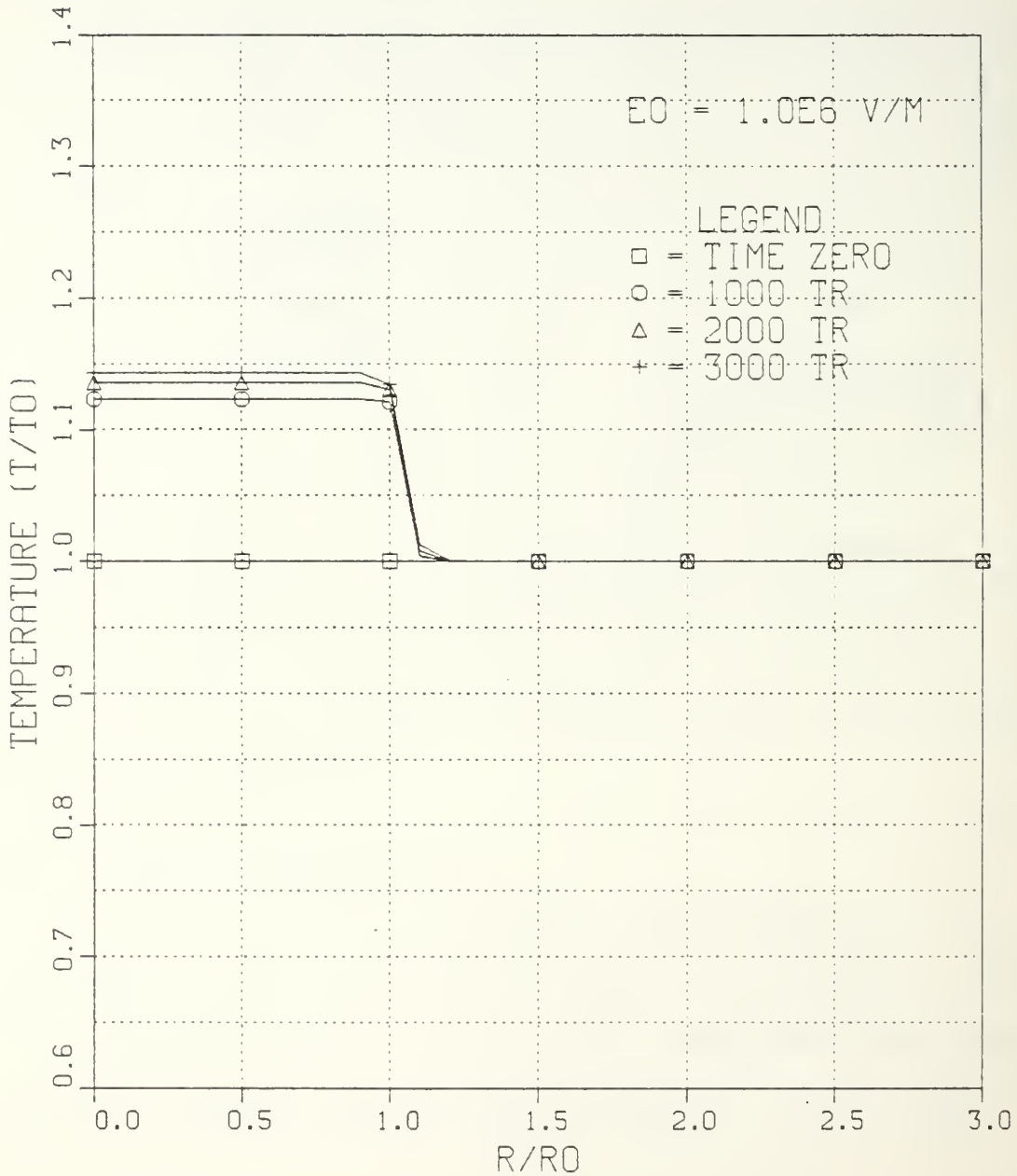

FIGURE 3.12

The temperature effects are then determined by Equation (2.30) where $\hat{n}$ is obtained by solving Equation (3.4). The first question in this case might be; what effect does turbulence have on the temperature profile due to this steady glow discharge? At sub-atmospheric pressure, it is known that the charged particle density profile takes on a Bessel function distribution [Ref. 9]. Therefore, to investigate this situation, Equation (2.30) can be solved independently from Equation (3.4) by calling up the subroutine "DENSTY" in the program and sending it a constant, for all time, value for $\hat{n}$. The Bessel function serves as the initial condition profile for the value of $\hat{n}$. This distribution, labeled "N1" in the program, allows $\hat{n}$ to go to zero at r = 4.8. Figure 3.13 shows the results of such a case for strictly laminar flow. The values of the controlling parameters for Figures 3.13 through 3.16 are: n_o = 3.125E18 m^{-3}, α_2 = 2.0E-12 m^3/sec and E_o = 1.0E6 V/m. It is clearly evident that the temperature for such a laminar case would rise to approximately 5400°K (recall that T/T_o = 1 is equivalent to 300°K) in only about 1000TR. Since molecular nitrogen (N_2) begins to dissociate at about 4000°K, the situation depicted in Figure 3.13 is undesirable. However, introducing turbulence into the flow tends to significantly reduce this temperature distribution. Figure 3.14 shows that a moderate degree of turbulence can be selected which, essentially, holds the temperature profile near the laminar profile without

REPRODUCED AT GOVERNMENT EXPEN

TEMPERATURE PROFILE

FOR BAMIN = 800

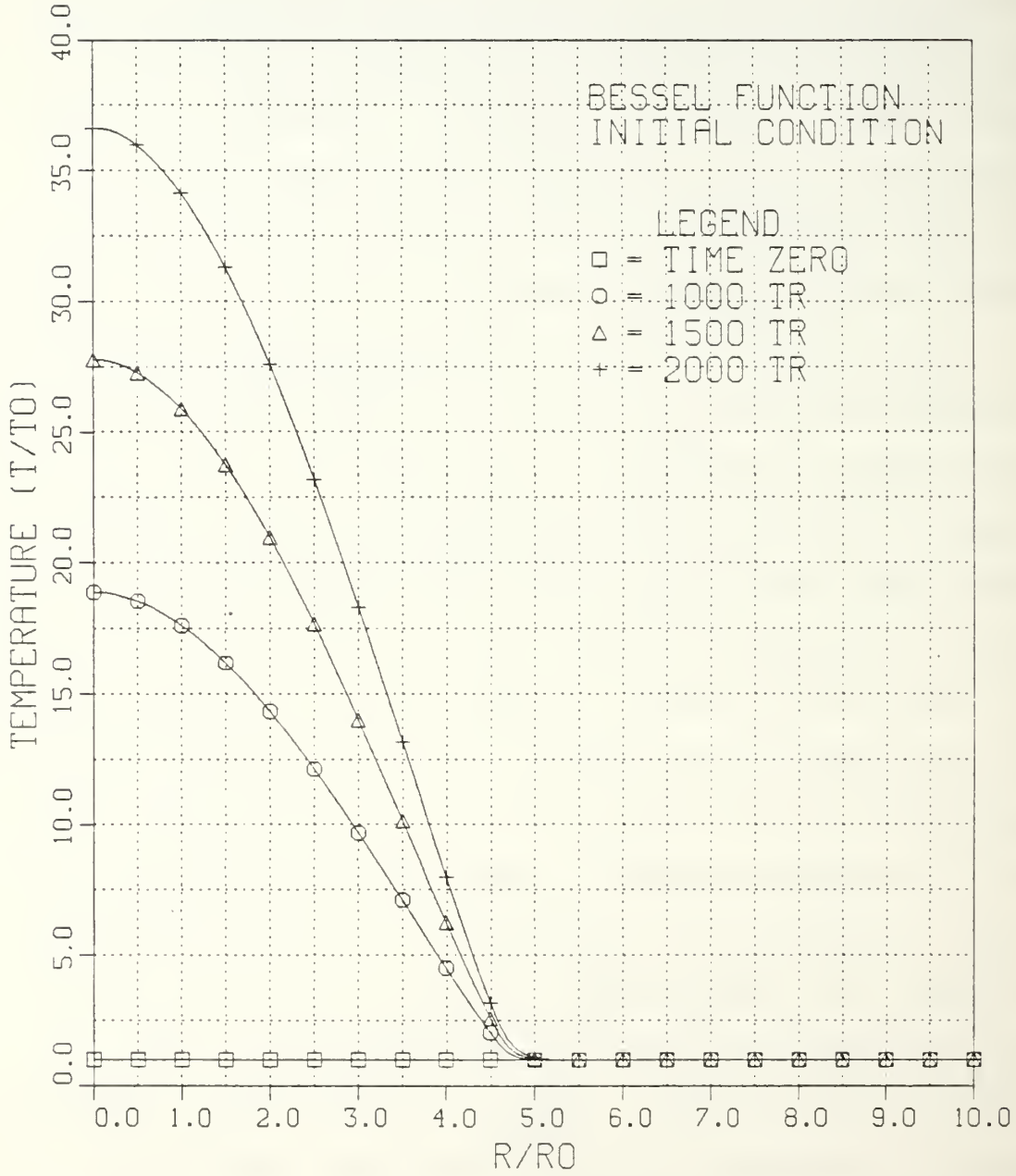

FIGURE 3.13

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

FOR BAMIN = 3

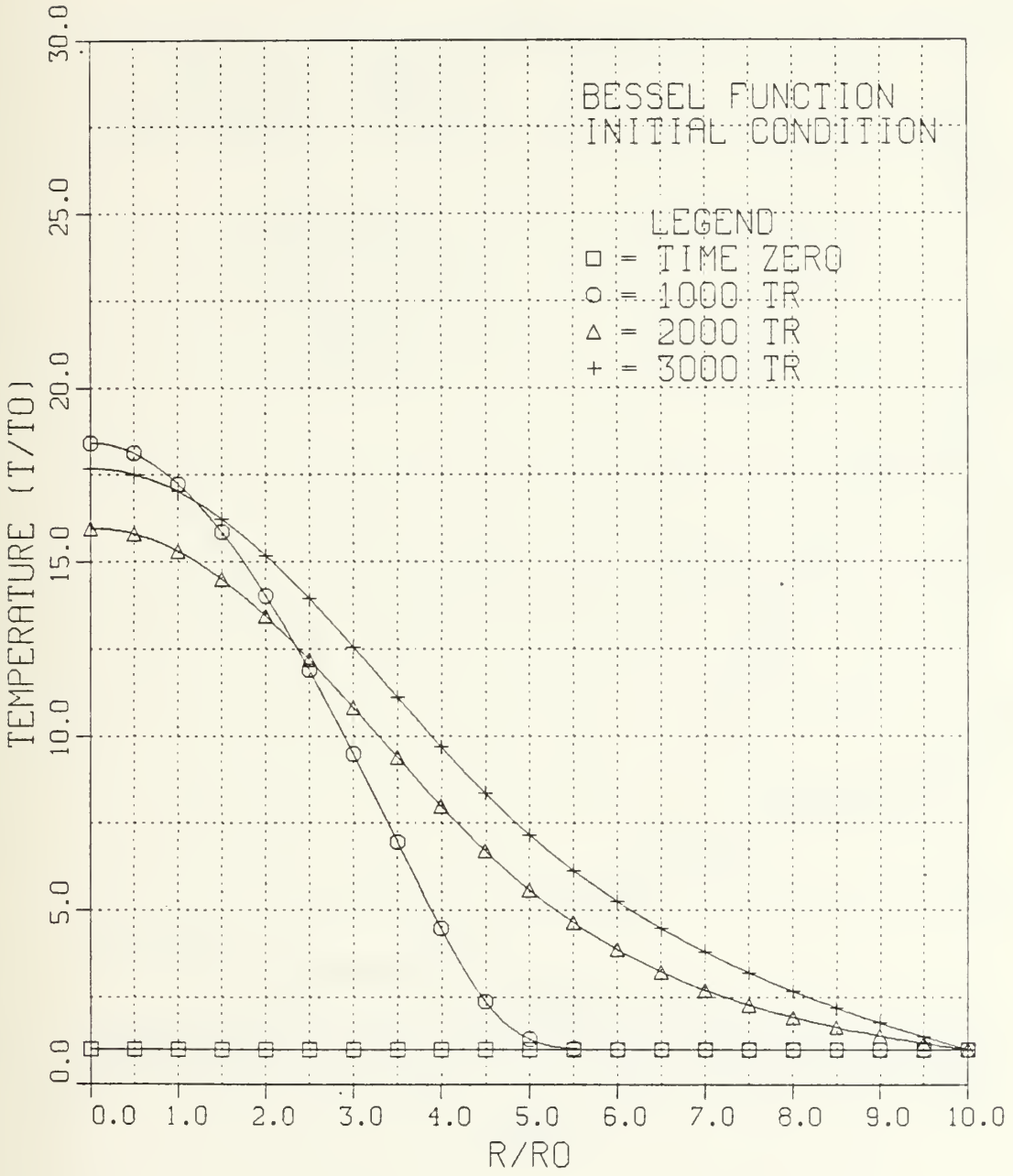

FIGURE 3.14

REPRODUCED AT GOVERNMENT EXPEN

TEMPERATURE PROFILE

FOR BAMIN = 1

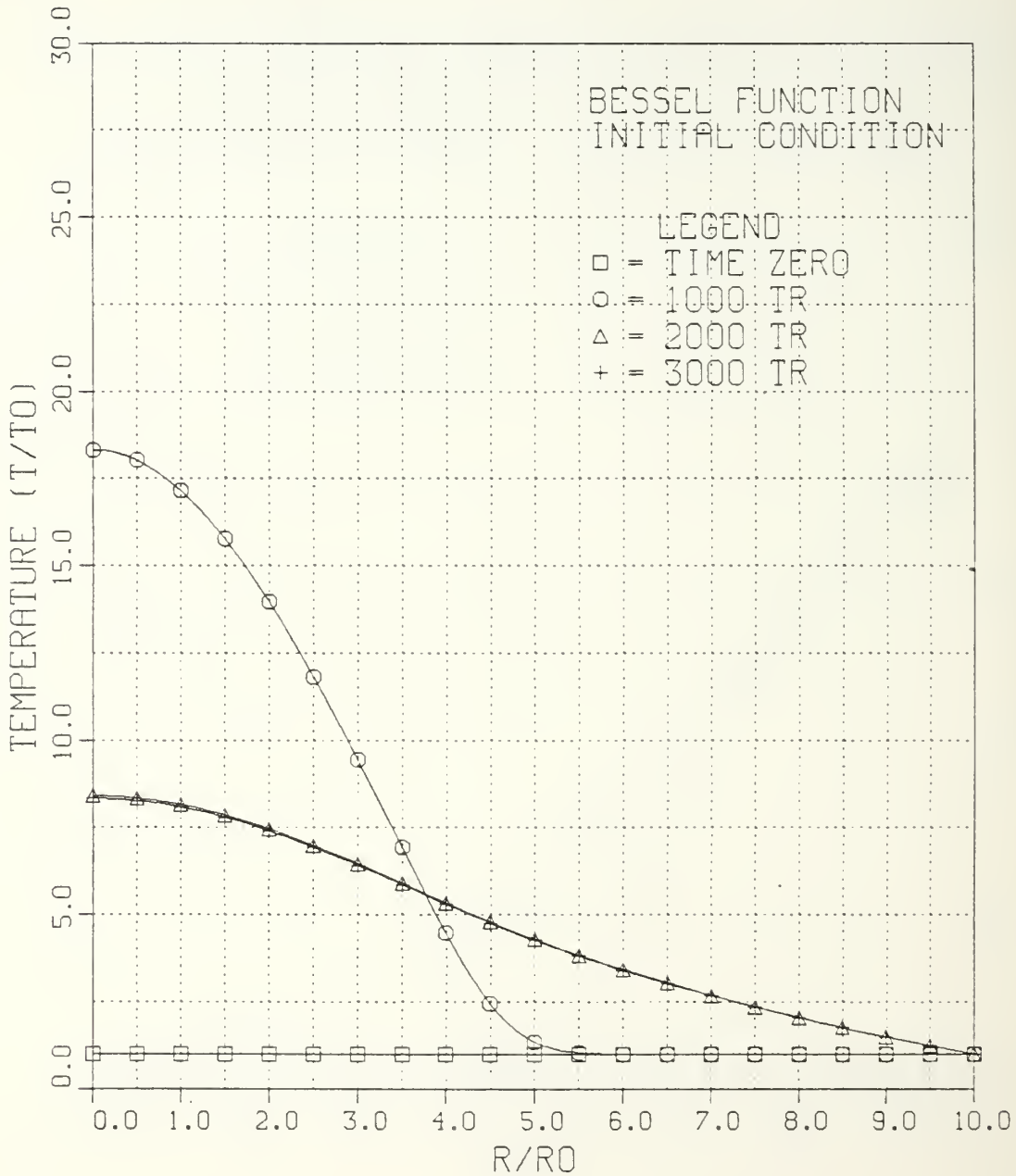

FIGURE 3.15

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

FOR BAMIN = 0.4

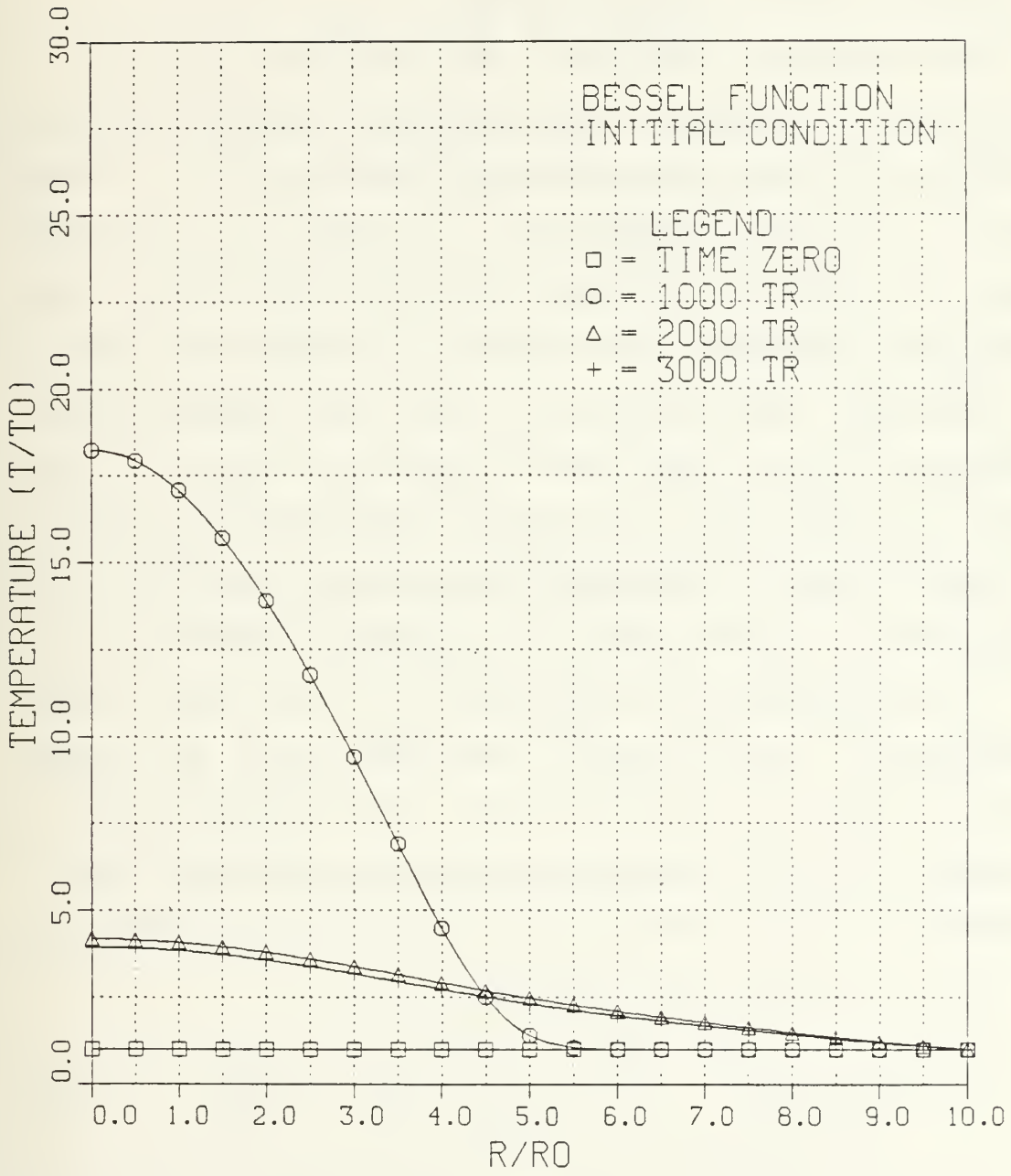

FIGURE 3.16

going unstable as in Figure 3.13. Figures 3.15 and 3.16 show the effects of increasing the turbulence level still further. Note that in Figure 3.15, the temperature profile is nearly "frozen out" at about 2400°K, while Figure 3.16 is holding steady at around 1200°K.

This asymptotic behavior of the temperature rise in reaching this stable profile appears to be related to the characteristic time associated with Equation (2.30). Observe that for all three figures (3.14, 3.15 and 3.16) the temperature rise stabilizes between about 2000TR and 3000TR. This effect will continue to be evident in the remaining cases to be discussed. The parameters for the case presently being considered are such that the true time of 2500TR is 0.4 msec. This is approximately 4.5 times the characteristic time step in seconds for the energy equation, which is consistent with the concept of a rise time in an asymptotic system.

The final case to be discussed here is perhaps the most interesting. This will be the case where there is a steady level of ionization in the gas and a perturbation of the electric field is introduced to model the effects of the streamer. For the steady state situation at the centerline, $\frac{\partial n}{\partial t} = 0$, so one can write Equation (3.5).

$$- \frac{D_a}{r} \frac{\partial}{\partial r}\left(r \frac{\partial n}{\partial r}\right)_{\ell} = 0 = \nu_i n_{\ell} - \alpha_2 n_{\ell}^2 - \alpha_3 n_{\ell}^3 \qquad (3.5)$$

Neglecting the three-body recombination term (α_3 is on the order of 1.0E-34 m^6/sec), and noting that $\frac{\partial n}{\partial r}$ on the centerline

is also zero, the ionization term is then equal to $\alpha_2 n_{\mathfrak{C}}$. Since Figure 2.2 gives ν_i/N, the charged particle density for the steady state condition can be obtained by,

$$n_{\mathfrak{C}} \approx \frac{\nu_i}{\alpha_2} = (\frac{\nu_i}{N})(\frac{N}{\alpha_2}) \qquad (3.6)$$

where N is given by Equation (2.28). Figure 3.17 shows this steady state charged particle distribution where, for this model, the glow discharge is contained in a region of a radius equal to 10 r_o. At r_o, the boundary condition is set such that $\hat{n}$ is zero at the "wall." The left hand boundary condition assumes that the slope of the charged particle density is zero. See Appendix A for details concerning how the program meets these boundary conditions. Note that in Figure 3.17, the steady state condition is reached in less than 100TR, and is constant at the longer times. The first vertical line, at $r/r_o = 1.0$, is the initial condition distribution which assumes $\hat{n} = 1.0$ out to $\hat{r} = 1.0$ and zero beyond this point.

The steady state solution for Equation (2.30) is given in Equation (3.7).

$$\hat{T} = -(\beta_A \hat{n}) \frac{\hat{r}^2}{4} + \beta_A \frac{\hat{n}\hat{b}^2}{4} + \hat{T}_o \qquad (3.7)$$

The boundary conditions are $\hat{T} = T_o$ at $\hat{r} = b$ and $\frac{\partial \hat{T}}{\partial \hat{r}} = 0$ at $\hat{r} = 0$. Figure 3.18 shows this result for $E_o = 1.47E5$ V/m. For this case $\beta_A = 0.2219$, and so at the centerline $\hat{r} = 0$

REPRODUCED AT GOVERNMENT EXPENS

CHARGED PARTICLE DENSITY PROFILE

WITH IONIZATION TERM

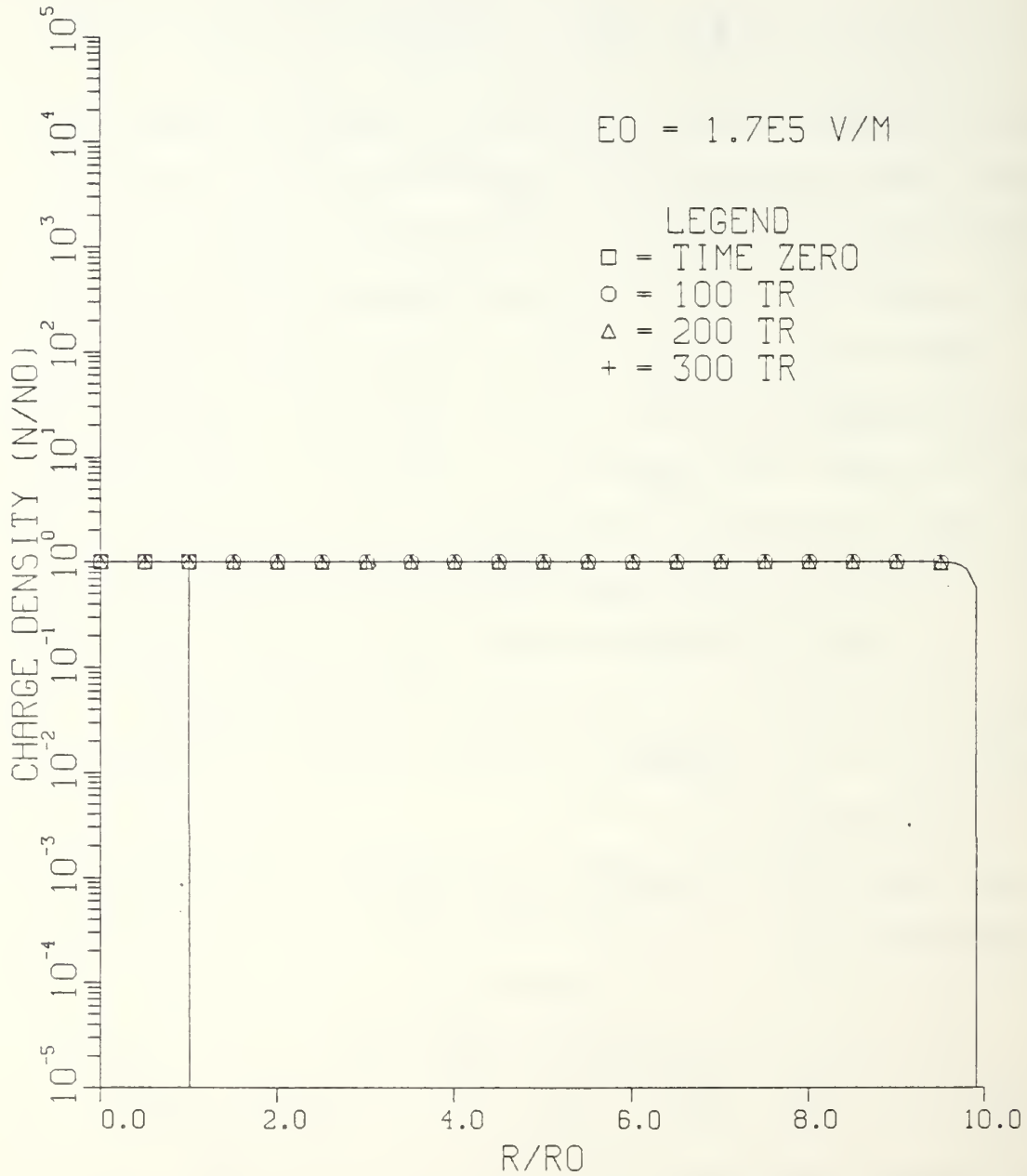

FIGURE 3.17

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

IN TURBULENT FLOW

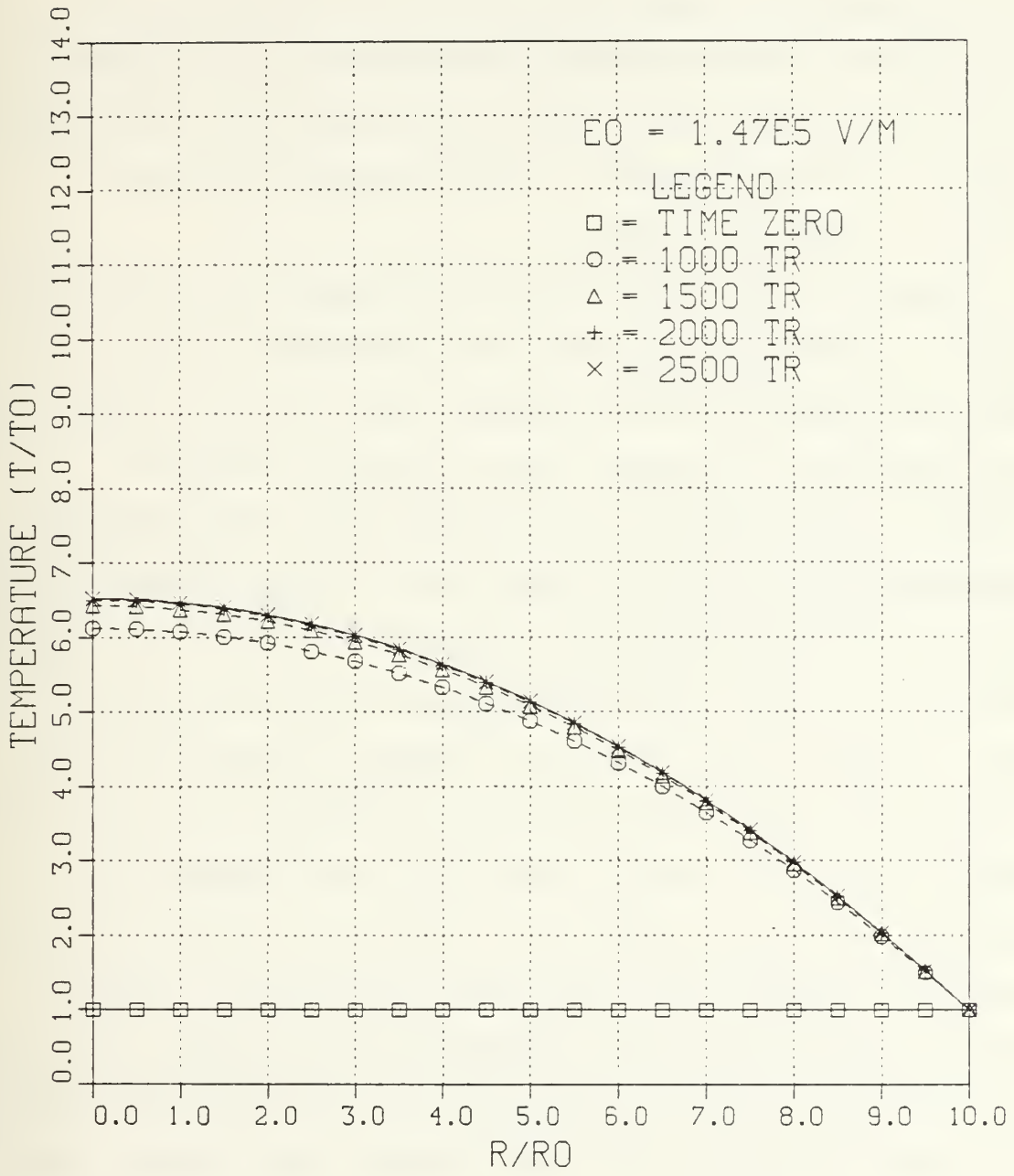

FIGURE 3.18

and substituting in the appropriate values into Equation (3.7) one obtains $\hat{T} = 6.548$ which is confirmed at $\hat{r} = 0$ in Figure 3.18. Note that this steady state condition is reached at approximately 2500TR which is about the same time period required for stabilization from the previous case for the Bessel function initial condition profile (Figures 3.14, 3.15 and 3.16) and for the case showing the temperature profile of a lone streamer in a non-ionized environment (Figures 3.6 through 3.12).

Figures 3.19 through 3.28 show the results of introducing the electric field perturbation into the model. Each plot shows the situation of the streamer appearing in a steady state turbulent environment and lasting for increasing periods of time beginning at 5 recombination times and being incremented by 5TR up to a final value of 50TR. In real time, this would be equivalent to 0.6 msec to 6.0 msec. For these plots, E_o = 1.47E5 V/m which produces an E/N of 6.0E-17 V-cm^2. This case was chosen for example purposes since the steady state temperature profile is convenient at a centerline value of $T/T_o = 6.5$ and the rise due to the perturbation reaches a maximum of about 13.75 (4125°K) at 50TR. Each figure shows five curves, 3 are solid and 2 are dashed. The solid lines represent the initial temperature distribution ($\hat{T} = 1.0$), the steady state profile, and then, the profile of the streamer at the end of its stay time. The dashed lines are the decaying, or transient, profiles produced after the appearance of

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

WITH STREAMER ON 5 TR

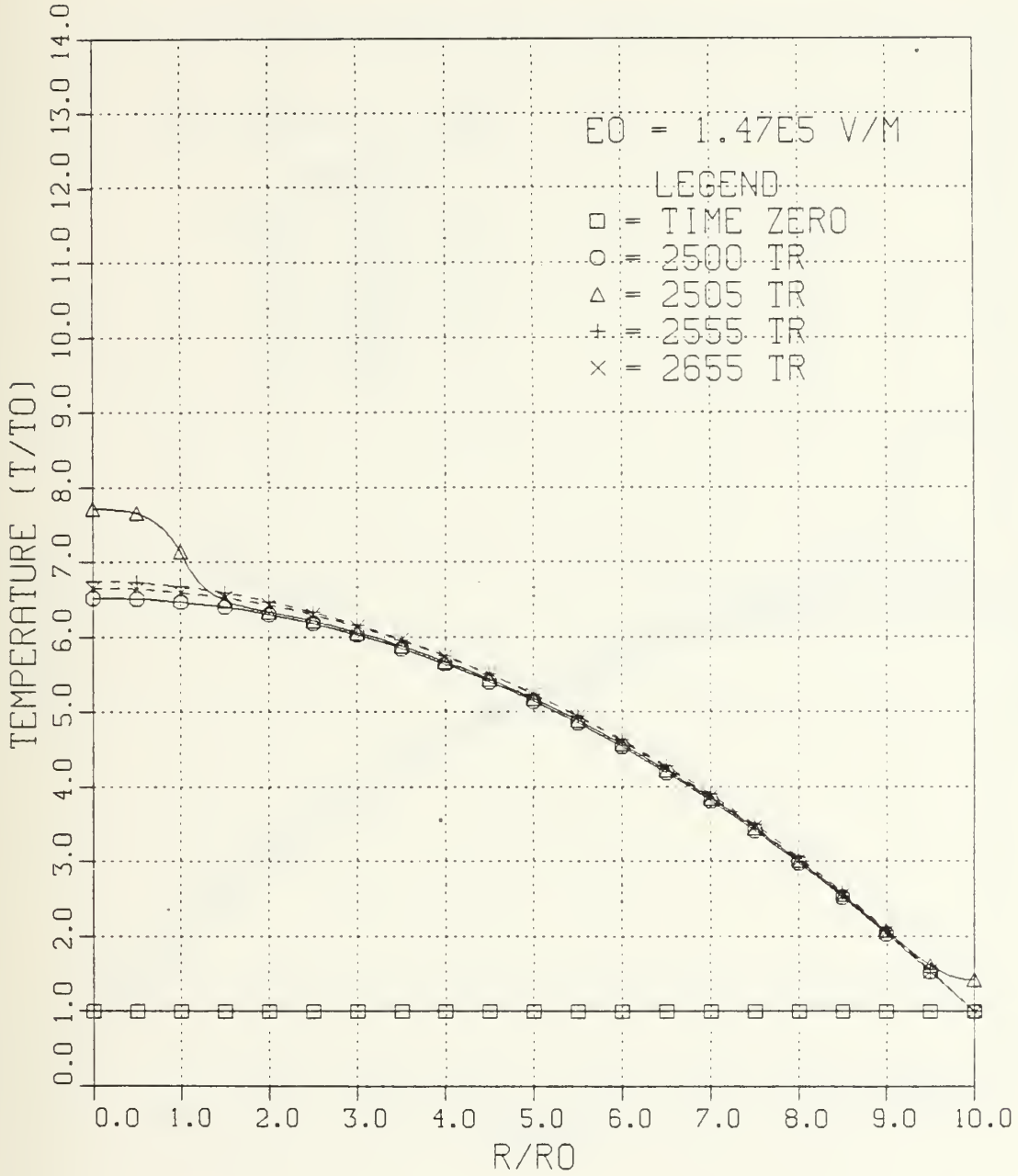

FIGURE 3.19

REPRODUCED AT GOVERNMENT EXPENS

TEMPERATURE PROFILE

WITH STREAMER ON 10 TR

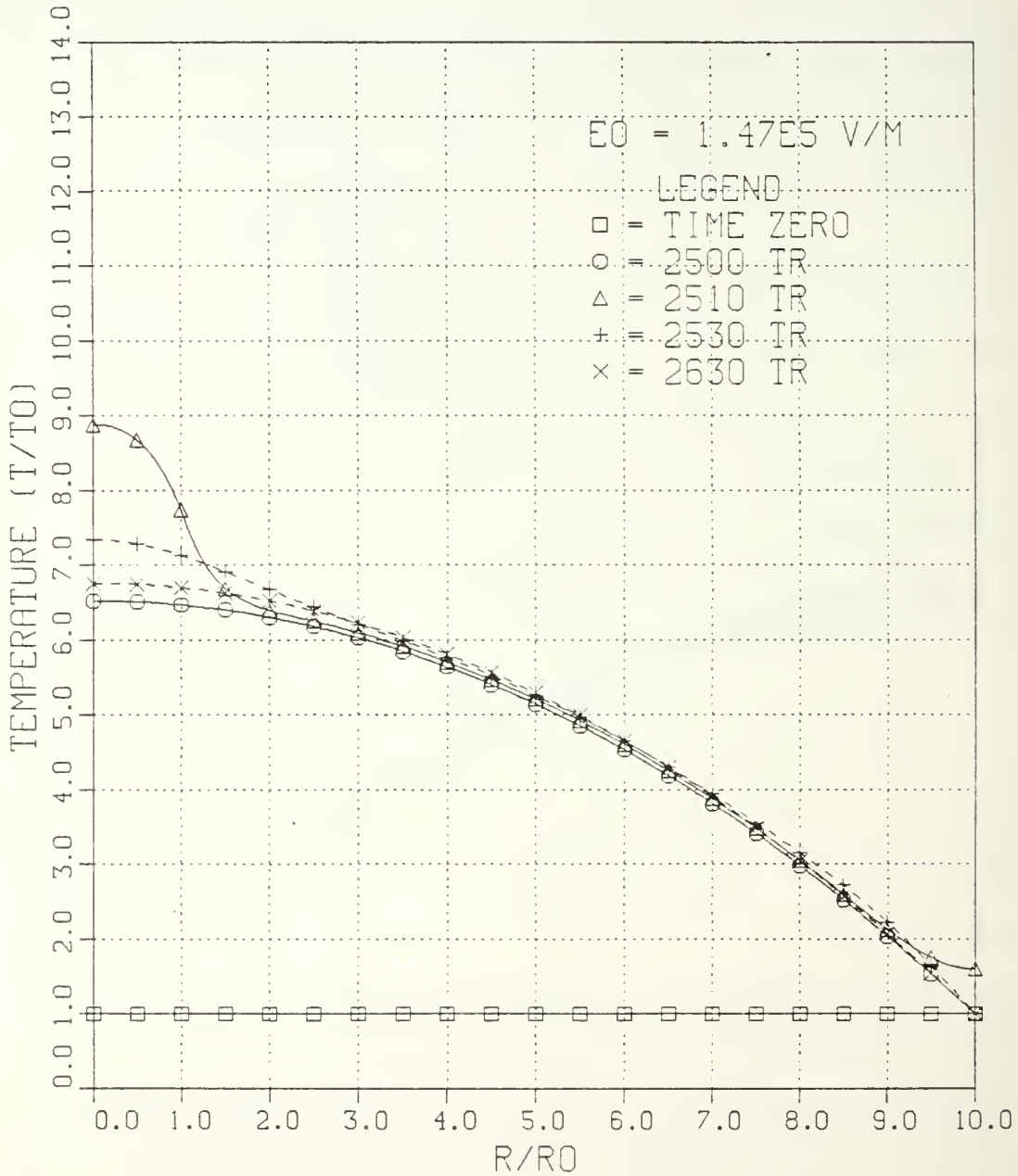

FIGURE 3.20

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

WITH STREAMER ON 15 TR

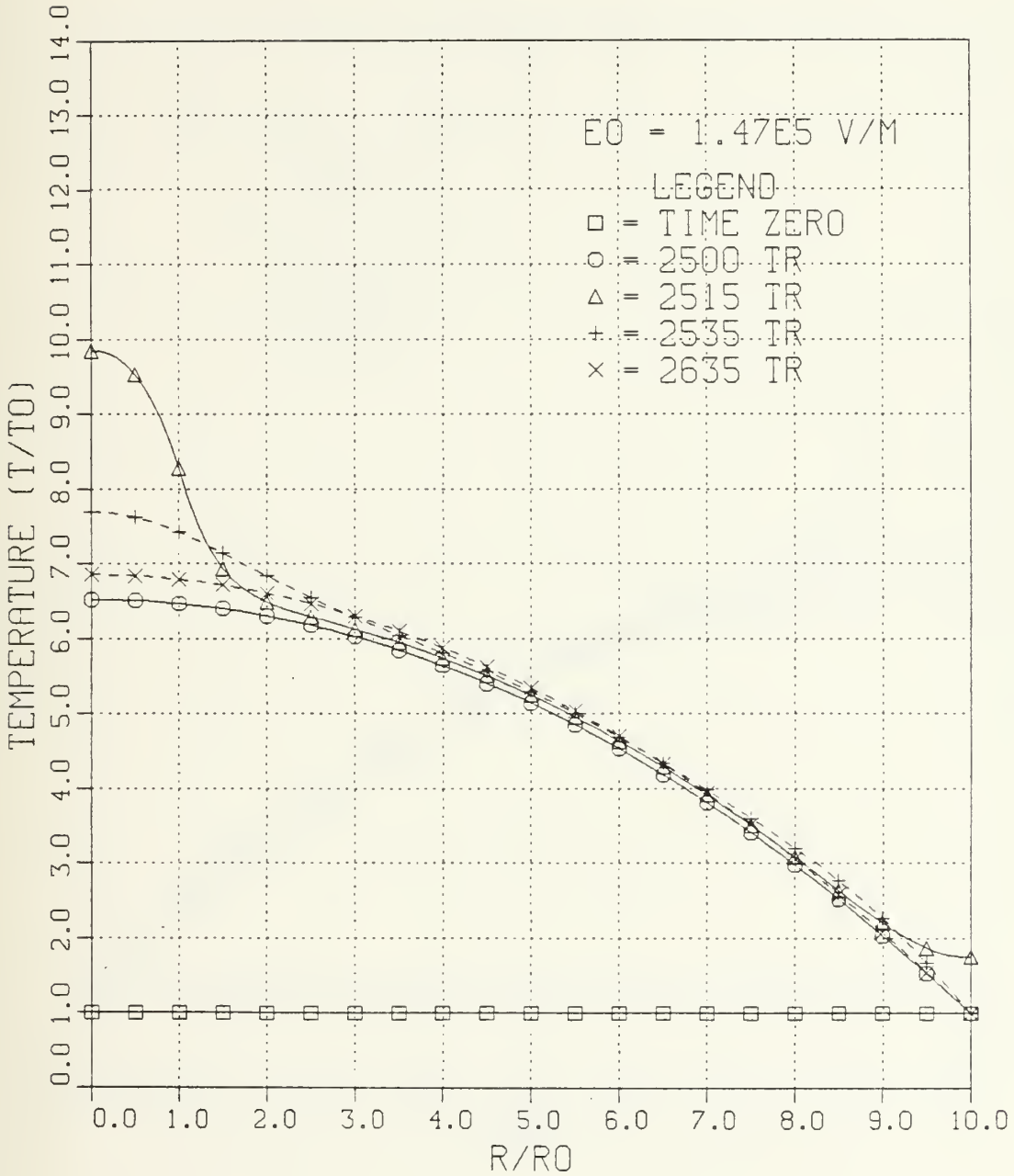

FIGURE 3.21

REPRODUCED AT GOVERNMENT EXPENS

TEMPERATURE PROFILE

WITH STREAMER ON 20 TR

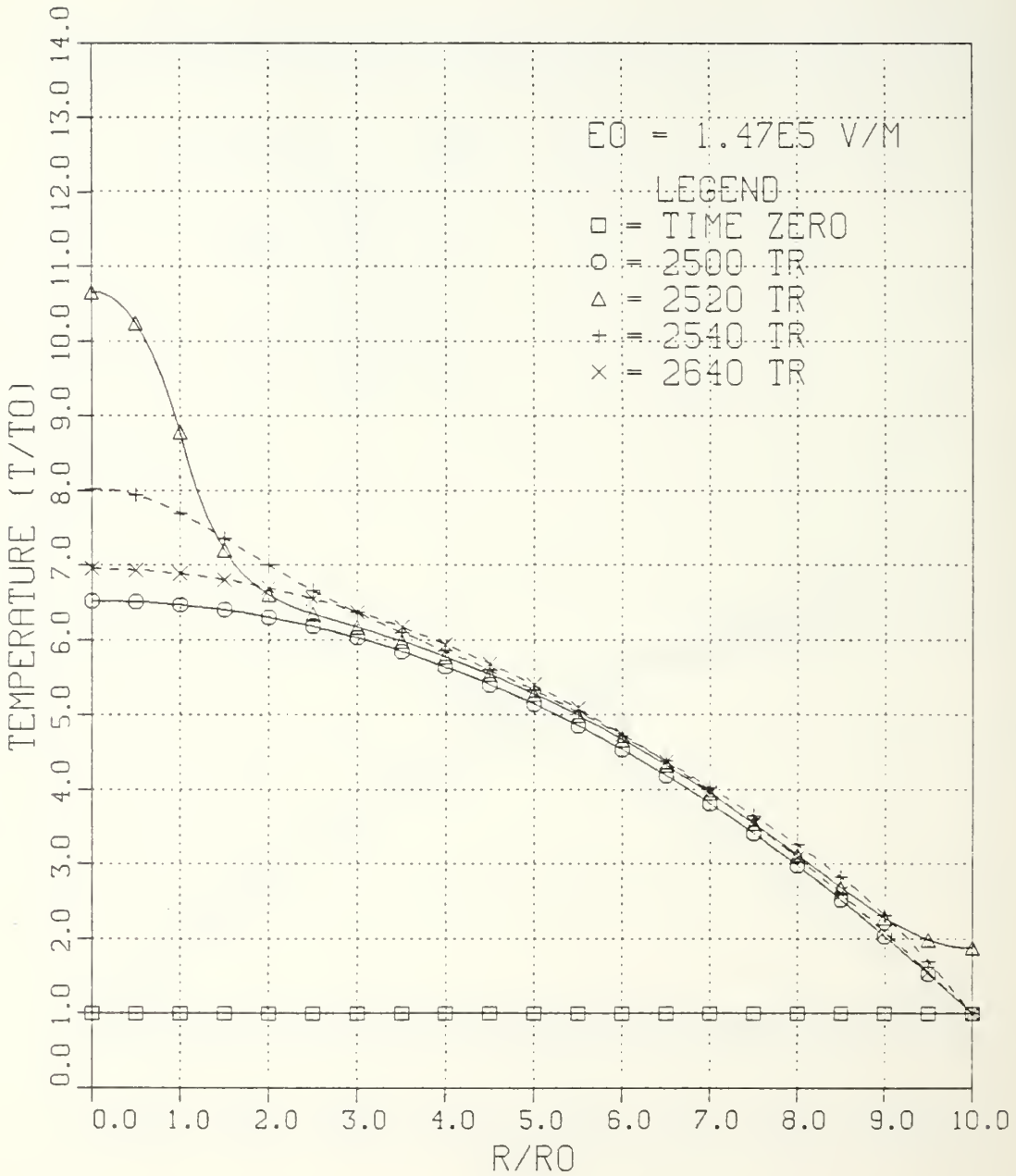

FIGURE 3.22

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

WITH STREAMER ON 25 TR

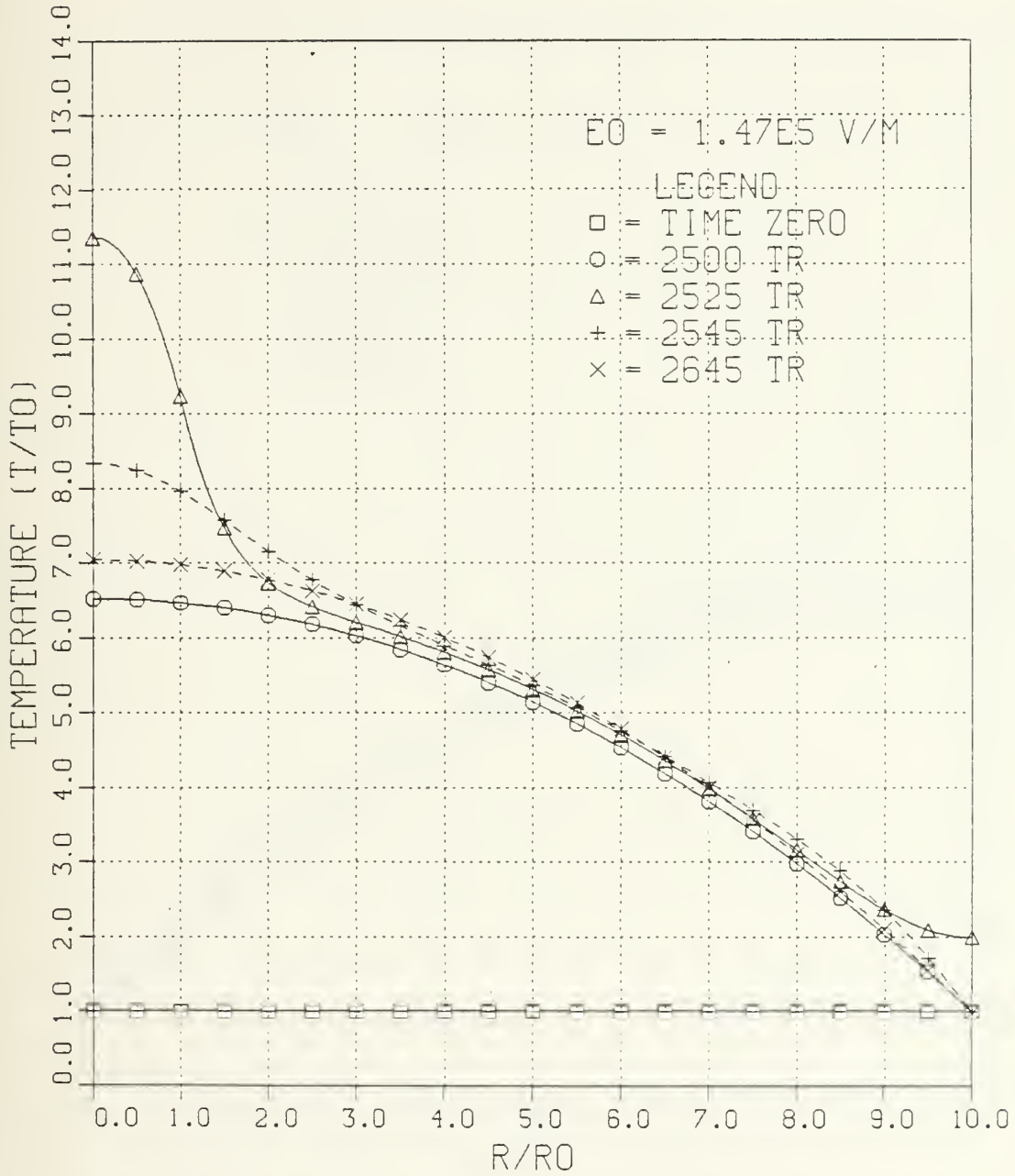

FIGURE 3.23

REPRODUCED AT GOVERNMENT EXPEN

TEMPERATURE PROFILE

WITH STREAMER ON 30 TR

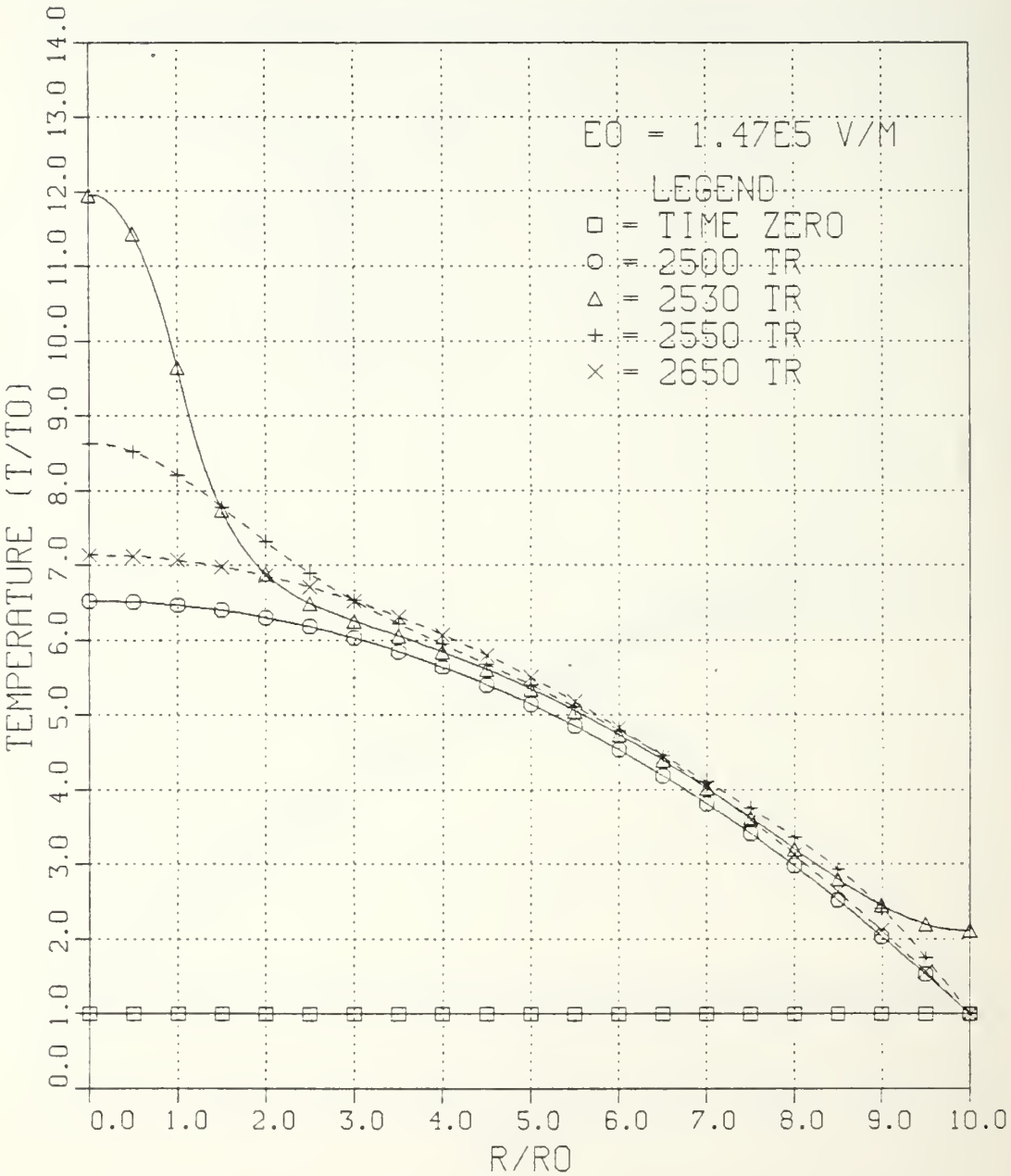

FIGURE 3.24

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

WITH STREAMER ON 35 TR

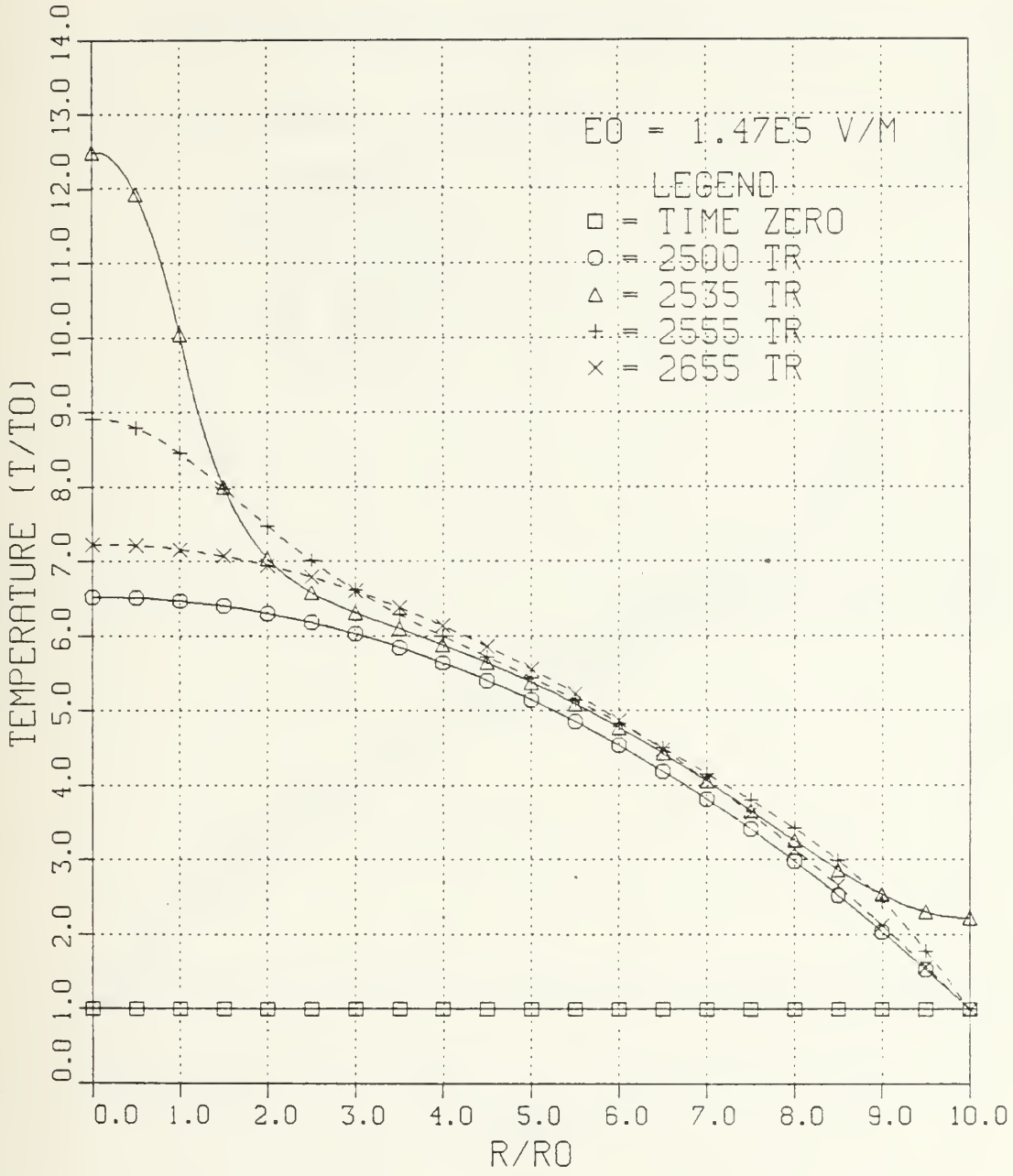

FIGURE 3.25

REPRODUCED AT GOVERNMENT EXPENS

TEMPERATURE PROFILE

WITH STREAMER ON 40 TR

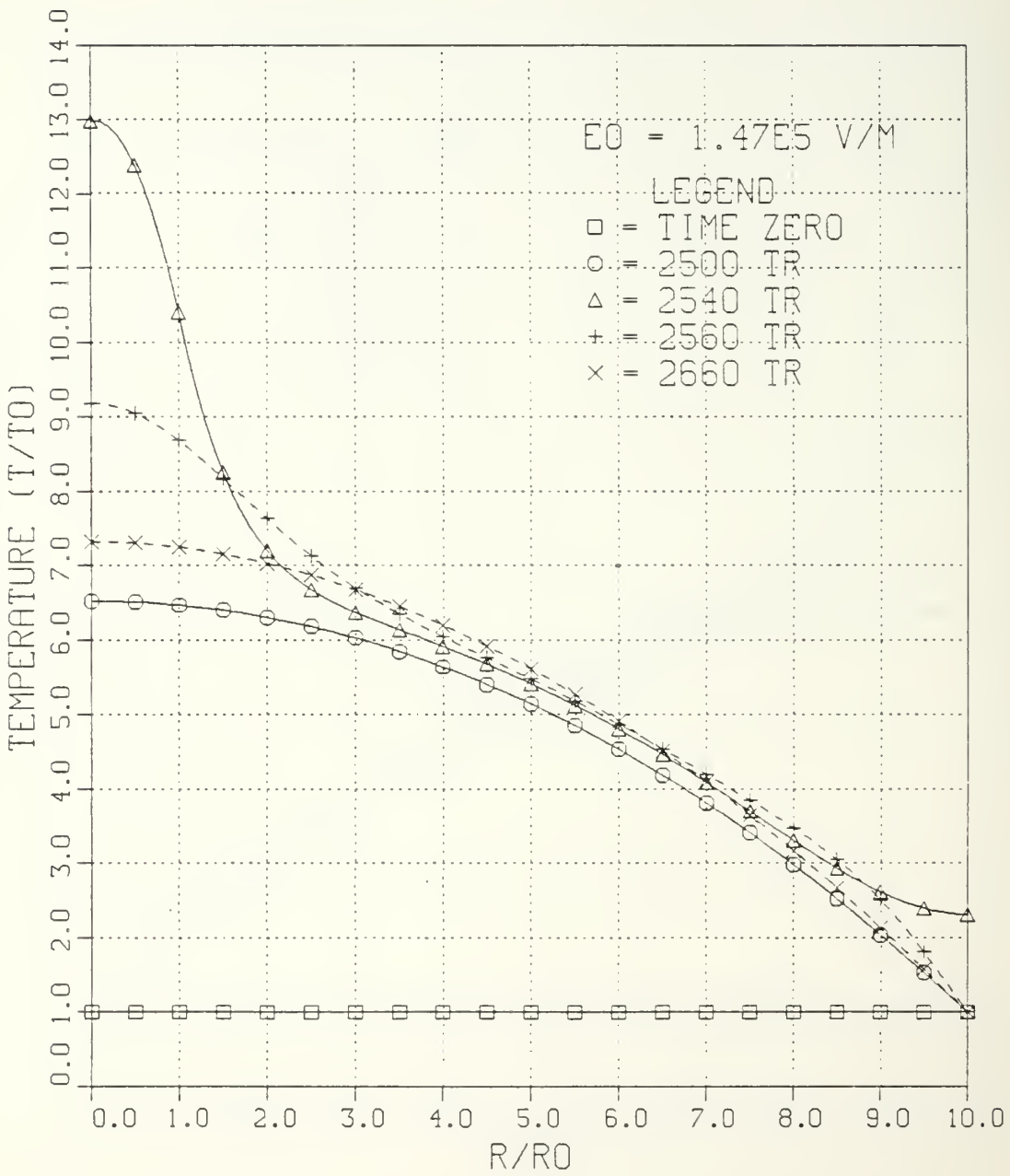

FIGURE 3.26

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

WITH STREAMER ON 45 TR

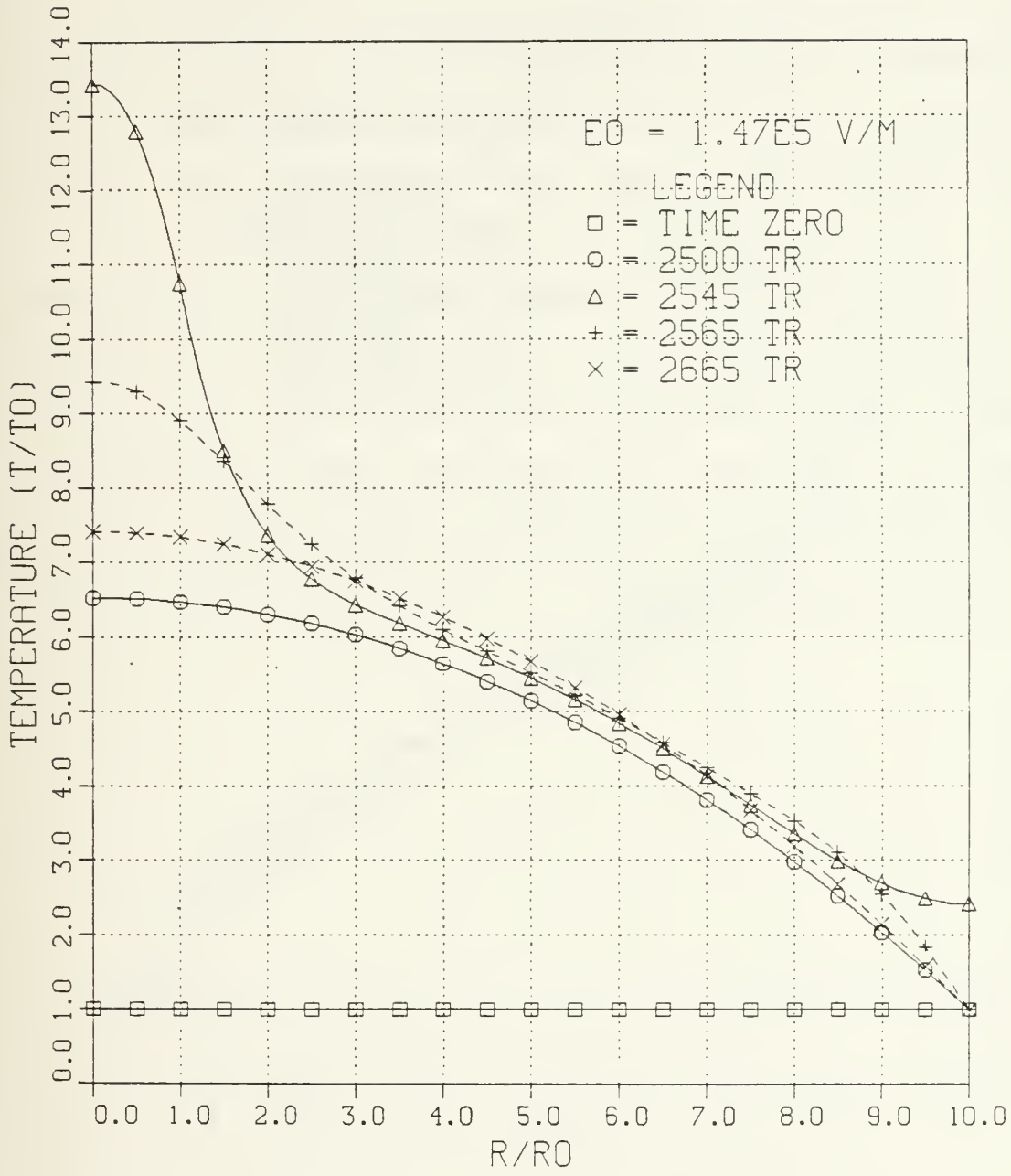

FIGURE 3.27

REPRODUCED AT GOVERNMENT EXPENS

TEMPERATURE PROFILE

WITH STREAMER ON 50 TR

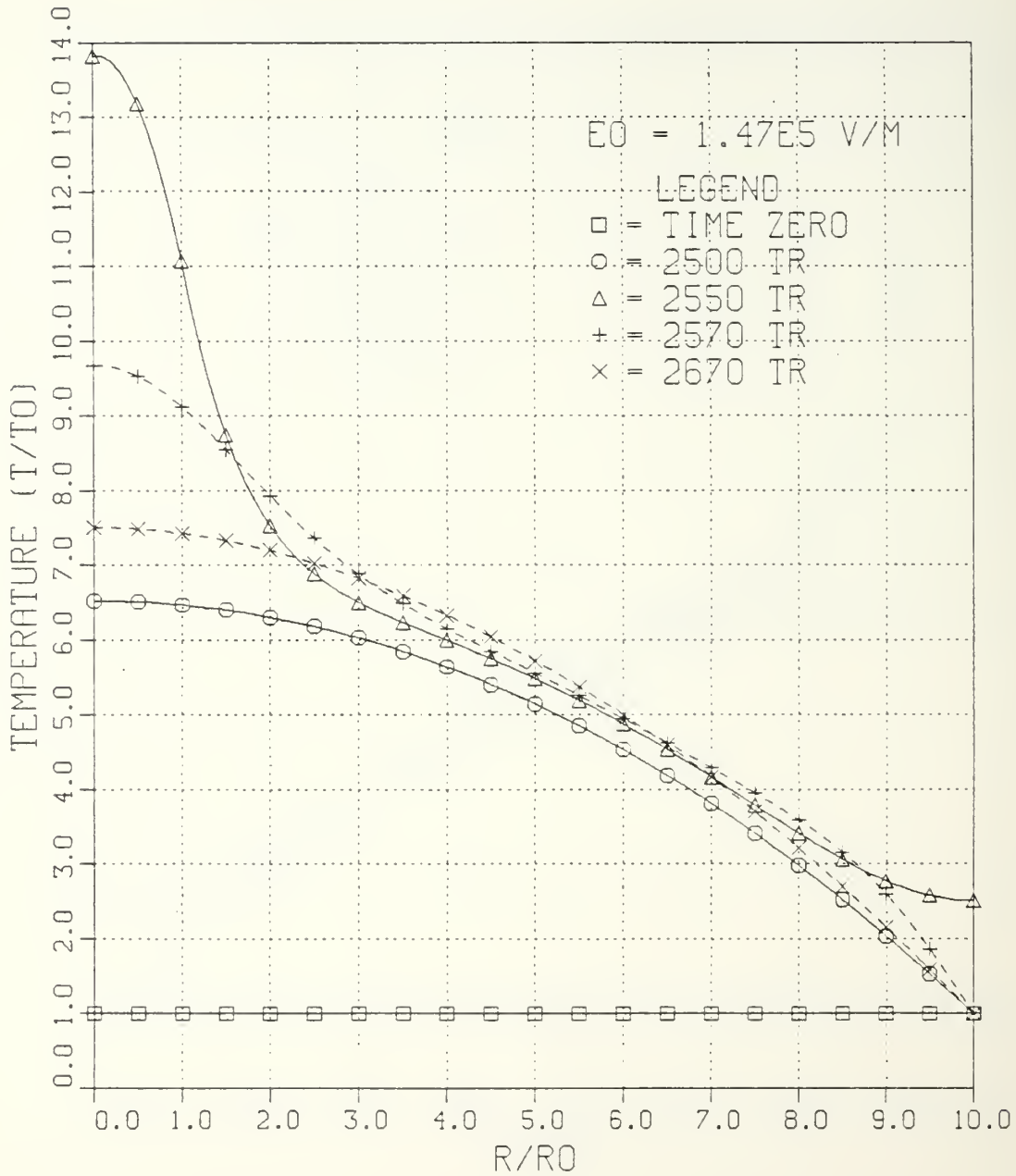

FIGURE 3.28

the streamer. Note that the centerline temperature drops to about half of the rise value in only 20TR and has nearly reached steady state in another 100TR.

Figures 3.29 and 3.30 show a case for a higher E/N value of 7.0E-17 V-cm^2. Here the steady state temperature profile has a centerline value of about 10.0 and then a streamer "on time" of only 8TR sends the profile above 4000°K. Cases for smaller E/N values produce lower steady state temperature profiles. The streamer effects are similar except that much longer "on times" are needed to produce much of a temperature rise. This appears consistent in the model since at the lower temperature it would take a much larger perturbation to upset the steady state condition, while at the higher energy levels a comparatively smaller perturbation is all that is required.

REPRODUCED AT GOVERNMENT EXPENS

TEMPERATURE PROFILE

WITH STREAMER ON 5 TR

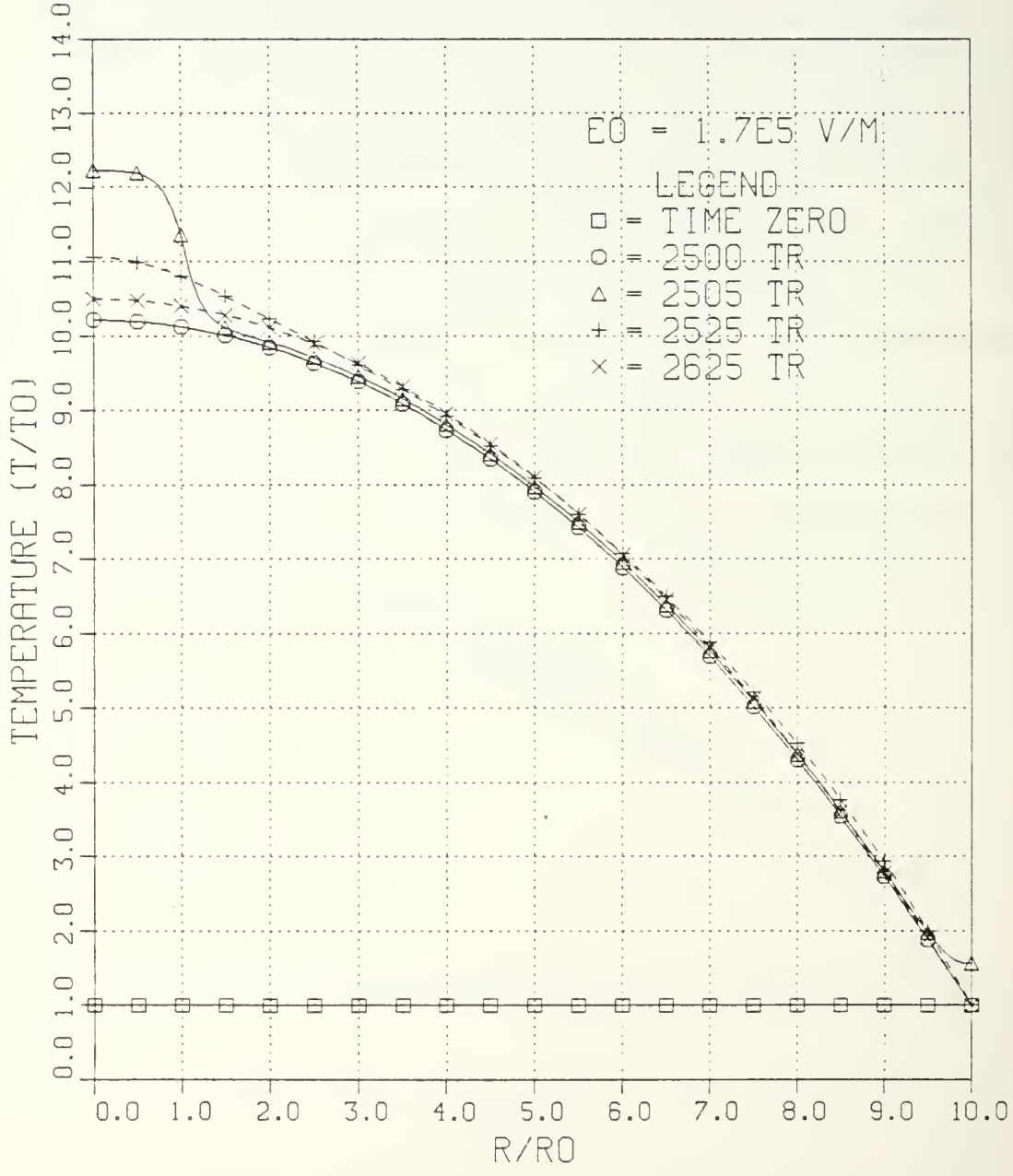

FIGURE 3.29

REPRODUCED AT GOVERNMENT EXPENSE

TEMPERATURE PROFILE

WITH STREAMER ON 8 TR

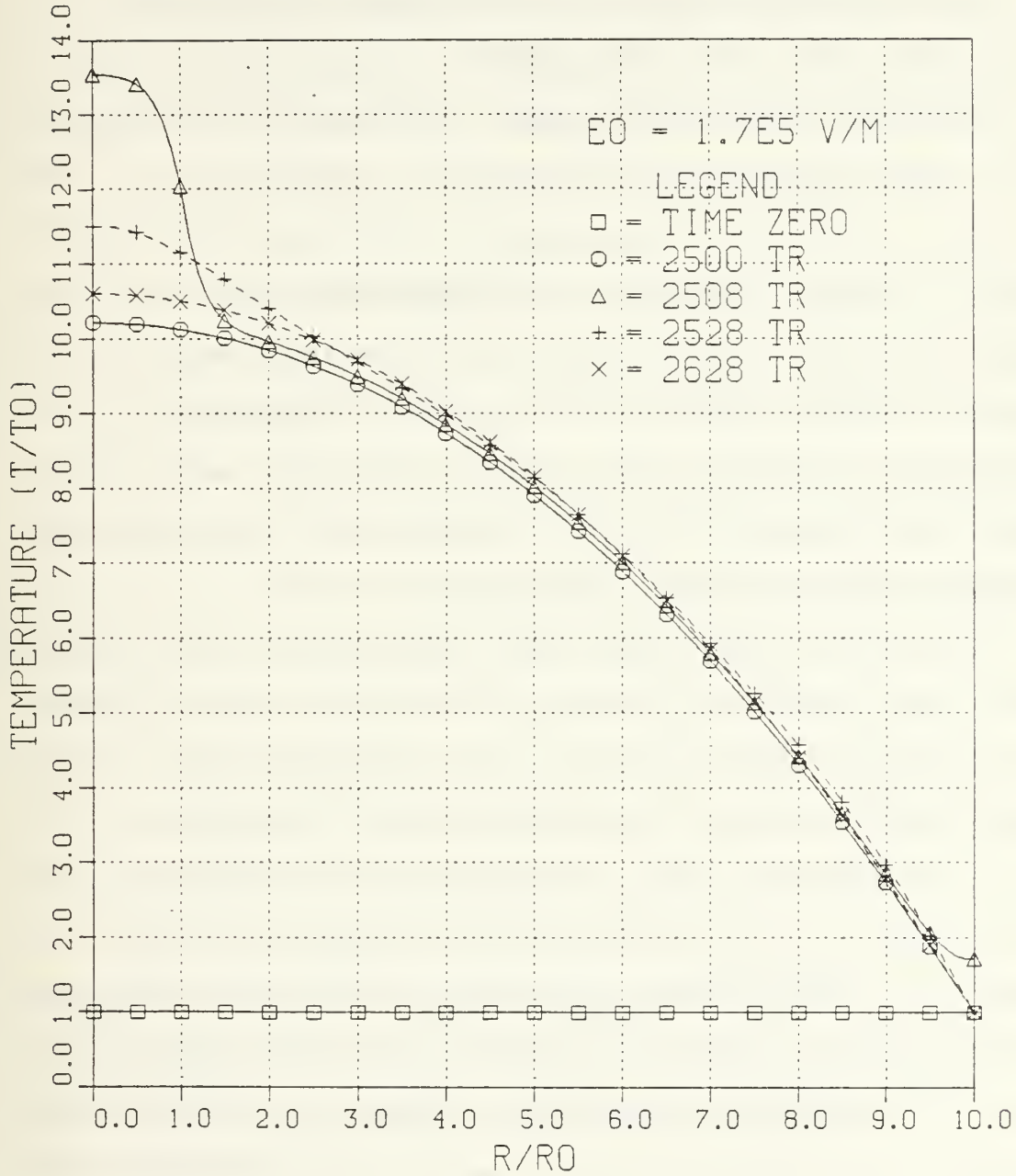

FIGURE 3.30

IV. CONCLUSIONS AND RECOMMENDATIONS

This thesis presents a mathematical model for describing the effects of varying levels of turbulence on glow discharge phenomena. The development is concerned only with the aftereffects of the streamer, rather than its propagation. The program that is presented in Appendix A is intended to allow the investigation of a large variety of cases of interest relating to the glow discharge. The cases discussed in the results chapter are those that appeared to develop a logical sequence of thought in attempting to answer the question of how turbulent gas flows can and do affect the stability of a glow discharge and the problem of delaying the glow-to-arc transition.

The results clearly show that the modelled turbulent flow does indeed have strong beneficial effects on the stability of the system. Additionally, the mathematical model successfully predicts within a reasonable level of uncertainty a region of stability that is consistent with the normal operating ranges of, say, an electrically pumped gas laser [Ref. 10]. The model suggests that the region for which the system maintains thermal stability is critically dependent on E/N. The research showed that a steady state temperature increase from 1200°K to 3000°K results from an increase in E/N of only 5.0E-17 V-cm^2 to 7.0E-17 V-cm^2.

One important question still remains however. That is; is there a particular level of turbulence which would have the most beneficial effect on the streamer-initiated breakdown problem? This work suggests that the best situation is simply that the most desirable level is the highest level that can be maintained. However, the non-linear nature of the equations suggests that some optimal level may exist [Ref. 2]. Experimental work is probably the best means of answering this question.

The results presented here by no means exhaust the aspects of this problem that could be investigated. The intent of the program is that it can be used as a tool to analyze any desired case. Some logical next steps might be to expand on the streamer modeling aspects to investigate what happens when a follow-on streamer appears before the initial one has fully decayed back to the steady state condition. The "Z" and "Z_A" arrays could be set up to have two or more streamers appear at any desired location. Also, the values of "Z" and "Z_A" used in this model are somewhat arbitrary and probably have to be specified for each individual E/N case according to experimental observation. More research into the actual magnitude of the electric field perturbation may allow the model to more accurately predict the areas of instability with streamer on times that are more in agreement with experiment [Ref. 5].

Mathematically, the analysis of this problem consists of solving the continuity equation for three cases. The first

is for the RHS of Equation (3.4) equal to less than zero. This is the non-ionized environment case. The second is for the RHS equal to zero. This is the steady state case which is what was applied to the ionized environment cases discussed. The third case, which was not applied in this thesis, is for the RHS of the continuity equation to be greater than zero. Investigation into this third case could possibly clarify some aspects of the problem. And finally, the energy equation may have eigenvalue solutions which might produce results of interest such as additional regions of stability.

LIST OF REFERENCES

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APPENDIX A

THE CHARGE PROGRAM

The Fortran program "CHARGE" is intended to be a working tool for analyzing the glow discharge problem. It evolved from a previously written program which solved the non-linear continuity equation (2.1) [Ref. 2]. It is now arranged in a somewhat different form. The program consists of a main program section which calls up any needed combination of the seven subroutines which perform the desired tasks. As currently written, it can compare four separate cases plus it will display the initial conditions of either the charged particle distribution or the temperature profile. Individual test cases are set up by commenting out (placing a "C" in column 1) those commands not desired. After the main program, there are two sections. The first provides commands to print out the data and the second provides commands to plot the results by utilizing the "DISSPLA" system developed by the Integrated Software Systems Corporation, San Diego, California.

The program as it appears following this discussion is set up to produce the plot displayed in Figure 3.28. The main program begins by defining all arrays. For this example the arrays are set to 101 in the "i" direction which specifies the size of the array with respect to radial position. Since one DR, one radial step, is equal to $r_o/10$,

position "101" specifies a distance of 10 r_o perpendicular from the centerline of the streamer. The "j" size is always kept at 2 due to the computing scheme used in the subroutines. After setting the array size, set the variable named "RADIUS" equal to the same value as the "i" coordinate above. In this case, RADIUS = 101. This determines how far in the radial direction that the differentiation is to procede. Next one must assign the desired values for BETA and BETAA depending on the case at hand. The BMIN, BMAX, BAMIN and BAMAX parameters apply when it is desired to transition the calculation from a laminar to a turbulent environment. This feature is designed to attempt to ensure that the calculation accounts for the fact that the streamer initially appears so quickly that the flow looks laminar to the streamer no matter what the level of turbulence is. Then, after a period of time, the charged particle distribution left behind by the streamer has existed for sufficient time to now "see" the turbulent effects. The time required for this transition is estimated by noting that the characteristic diffusion time for laminar flow is approximately 0.1 msec [Ref. 2]. Depending on the case at hand, one DT is on the order of a few microseconds. Then, 100 DT (10TR) is on the order of a few tenths of a millisecond. Therefore, for most cases, a transition of 100 DT is used.

Returning to the example, one procedes by selecting the desired initial conditions. In this example, all of the

"N1" values are commented out since they apply to the Bessel function initial condition profile. Four series of subroutine calls are provided then to allow running of four separate cases. Each case assigns its respective results to the "VALX" (X = 0,1,2,3,4...) array for plotting later.

The initial conditions are assigned to VALO or VALTO. Then the first subroutine call will see the initial distribution values of n while each successive call will see the results of the previous call. If it is desired to recover the initial conditions, call either "NZERO" or "TZERO" as desired and the current values will then be reset to time zero.

For this example now, "VAL1" is the result of two calls. The first is for "TRNSTY" to transition from laminar to turbulent conditions with time set to 101, then "DENSTY" is called and runs for 23,900 DT. The result is the steady state temperature distribution in turbulent flow at 2500TR. VAL2 is then the result of calling up the subroutine "STREMR" and running it for 500 DT with BETA and BETAA set to the laminar values. VAL3 then results from calling up "TRNSTY" for 100 DT and "DENSTY" for an additional 100 DT at the turbulent values. Finally, VAL4 simply runs "DENSTY" for another 1000 DT in turbulent conditions to check the decaying temperature profile. Of course, any combination of these subroutine calls and time values can be selected to view the situation as desired.

The results can then be printed out and/or plotted. To become familiar with the "DISSPLA" plot system, refer to DISSPLA Pocket Guide (current with version 9.0) by ISSCO Graphics, 10505 Sorrento Valley Rd, San Diego, California.

Some pointers of potential errors should be clarified. When selecting the desired heading lines, ensure that the last entry in the call to "HEADIN" correctly indicates the total number of heading lines. If no heading appears, this is probably the reason. Secondly, ensure that the third entry in the call to "CURVE" is equal to the "i" value on the arrays. This is the number of points (101 here) to be plotted. And finally, the second entry in the call to "LEGEND" is the number of lines of legend needed, 5, in this example.

The subroutines "DENSTY," "STREMR" and "TRNSTY" all contain essentially the same algorithm with modifications to perform their particular function. The basic idea here is that the calculation procedes from the first row of data and fills the second row with updated values. Then the first row of old data is replaced by the second row of new data by "SWITCH" and the loop increments one DT and procedes. With this scheme the program can be run as long as desired without using large values of memory storage.

The boundary conditions for the problems are met by the routine. The left boundary condition assumes that the slope is zero across the streamer centerline. So here, the new

value at $\hat{n}(2,2)$ is assigned to $\hat{n}(1,2)$ before the switch. The right boundary assumes that n is zero at the wall, that is station 101. This is met by limiting the maximum radius value to be one less than "RADIUS" (called "TEMRAD"). The routine procedes to "TEMRAD," then the last point seen by the calculation algorithm is zero. Similarly, the temperature of the right boundary is set to 1.0.

To compute the conductance parameter, gamma, that would result from any particular $\hat{n}$ distribution, call "CNDUCT" after assigning the desired $\hat{n}$ distribution to some "VALX." The single value "NN" will be returned, which is gamma.

It is hoped that the program is sufficiently self-explanatory. All variable, constant and subroutine names are defined and documentation comments are provided.

```
C     THIS PROGRAM MODELS THE EFFECTS OF TURBULENT GAS FLOW ON THE       CHA0C010
C     STABILITY OF A GLOW DISCHARGE BREAKDOWN STREAMER.  TO ACCOMPLISH   CHA0C020
C     THIS, THE PROGRAM CALCULATES THE CHARGED PARTICLE DENSITY AND      CHA0C030
C     TEMPERATURE PROFILES FOR AN ELECTRIC DISCHARGE IN A FLOWING GAS    CHA00040
C     ENVIRONMENT.  THE PROGRAM USES A FINITE DIFFERENCE SCHEME TO       CHA0C050
C     NUMERICALLY SOLVE A CONTINUITY EQUATION TO DETERMINE THE CHARGED   CHA0C060
C     PARTICLE DENSITY DISTRIBUTION THEN USES A TRAPIZOIDAL NUMERICAL    CHA00070
C     INTEGRATION SCHEME TO DETERMINE THE CONDUCTANCE.  ADDITIONALLY,    CHA0C080
C     THE PROGRAM SIMULTANEOUSLY SOLVES AN ENERGY EQUATION UTILIZING     CHA0C090
C     THE CALCULATED DENSITY VALUES TO DETERMINE THE TEMPERATURE         CHA0C100
C     PROFILE IN THE GAS IN THE IMMEDIATE REGION OF THE ELECTRIC         CHA0C110
C     DISCHARGE STREAMER.  AND FINALLY, TO MODEL THE STREAMER, A         CHA00120
C     MOMENTARY PERTURBATION OF THE ELECTRIC FIELD OVER A RADIAL         CHA0C130
C     DISTANCE FROM THE STREAMER CENTERLINE IS INTRODUCED AND THEN       CHA0C140
C     THE RESULTING REACTION OF THE SYSTEM IS OBSERVED.                  CHA00150
C     BOTH THE CONTINUITY AND THE ENERGY EQUATIONS HAVE BEEN NON-        CHA0C160
C     DIMENSIONALIZED.                                                   CHA00170
C                                                                        CHA0C180
C     W.R.OKER - APRIL 1984                                              CHA0C190
C                                                                        CHA00200
C     NAVAL POSTGRADUATE SCHOOL, MONTEREY, CALIFORNIA                    CHA0C210
C                                                                        CHA0C220
C ***************************************************                    CHA00230
C * DEFINITIONS OF VARIABLE AND CONSTANT NAMES *                        CHA0C240
C ***************************************************                    CHA00250
C                                                                        CHA0C260
C     N = NORMALIZED CHARGED PARTICLE DENSITY (N = N/NO)                 CHA0C270
C     NO = N AT TIME ZERO                                                CHA0C280
C     NN = RESULT OF INTEGRATION, EQUALS THE CONDUCTANCE WHEN            CHA0C290
C          MULTIPLIED BY A CONSTANT. EQUAL TO GAMMA IN THE TEXT.         CHA00300
C     N1 = BESSEL FUNCTION INITIAL CONDITION PROFILE FOR N/NO            CHA0C310
C     T = NORMALIZED TEMPERATURE (T = T/TO), TO = 300 DEGREES KELVIN     CHA0C320
C     DA = AMBIPOLAR DIFFUSION COEFFICIENT                               CHA00330
C     RADIUS = SPECIFIES RADIAL DISTANCE FROM STREAMER CENTERLINE        CHA0C340
C              FOR DIFFERENTIATION LIMIT                                 CHA0C350
C     R = RADIUS VALUE FOR EACH CALCULATION STEP                         CHA0C360
C     RO = RADIUS OF ELECTRIC STREAMER AT TIME ZERO                      CHA0C370
C     RHATO = "I" COORDINATE OF RO                                       CHA0C380
C     RHAT1 = RHATO + 1                                                  CHA0C390
C     R1 = RADIUS VALUE FOR EACH CALCULATION STEP USED BY CONDUCTANCE    CHA0C400
C          SUBROUTINE                                                    CHA00410
C     CONRAD = SPECIFIES RADIAL DISTANCE FROM STREAMER CENTERLINE FOR    CHA0C420
C              CALCULATING THE CONDUCTANCE OUT TO A SPECIFIED RADIUS     CHA0C430
C     TEMRAD = (RADIUS - 1) USED FOR TEMPERATURE PROFILE CALCULATION     CHA0C440
C              TO INSURE PROPER BOUNDARY CONDITION                       CHA0C450
C     A2 = TWO-BODY RECOMBINATION COEFFICIENT                            CHA0C460
C     A3 = THREE-BODY RECOMBINATION COEFFICIENT                          CHA0C470
C     TIME = DETERMINES NUMBER OF TIME STEPS TO BE USED FOR CALCULATION  CHA0C480
```

```
C      DT = DELTA T FOR DENSITY CALCULATION (TIME STEP)                    CHA0C490
C      DTA = DELTA T FOR TEMPERATURE CALCULATION                           CHA0C500
C      DR = DELTA R  (RADIUS STEP)                                         CHA0C510
C      ALPHA = NON-DIMENSIONALIZED RECOMBINATION COEFFICIENT               CHA0C520
C      BETA = FLOW CONDITION PARAMETER (DEGREE OF LAMINAR OR               CHA0C530
C             TURBULENT CONDITION) FOR DENSITY CALCULATION                 CHA0C540
C      BETAA = SAME AS BETA FOR TEMPERATURE CALCULATION                    CHA0C550
C      BMIN = BETA AT END OF TRANSITION TO TURBULENT CONDITIONS             CHA0C560
C      BMAX = BETA AT BEGINING OF TRANSITION FROM LAMINAR TO               CHA0C570
C             TURBULENT CONDITION                                          CHA0C580
C      BAMIN = BMIN FOR TEMPERATURE CALCULATION                            CHA0C590
C      BAMAX = BMAX FOR TEMPERATURE CALCULATION                            CHA0C600
C      RR = R/RO ARRAY (FOR PLOTTING)                                      CHA0C610
C      VAL'X' = N VALUE ARRAY (FOR PLOTTING)                               CHA0C620
C      VALT'X' = T VALUE ARRAY (FOR PLOTTING)                              CHA0C630
C      EO = INITIAL ELECTRIC FIELD STRENGTH (FOR REFERENCE, NOT USED.      CHA0C640
C           FOR CALCULATION PURPOSES)                                      CHA0C650
C      Z = MULTIPLIER FOR CHARGED PARTICLE DENSITY VALUE TO SIMULATE       CHA0C660
C          THE ELECTRIC FIELD PERTUBATION INTRODUCED BY THE STREAMER       CHA0C670
C      ZA = SAME AS "Z" BUT FOR TEMPERATURE VALUE                          CHA0C680
C                                                                          CHA0C690
C  ************************                                                CHA0C700
C  * LIST OF SUBROUTINES *                                                CHA0C710
C  ************************                                                CHA0C720
C                                                                          CHA0C730
C      NZERO = INITIALIZES CHARGED PARTICLE DENSITY PROFILE                CHA0C740
C      TZERO = INITIALIZES TEMPERATURE PROFILE                             CHA0C750
C      SWITCH = REPLACES "FIRST ROW" DATA WITH COMPUTED "SECOND            CHA0C760
C               ROW" DATA FOR COMPUTATION ALGORITHM SCHEME                 CHA0C770
C      DENSTY = CALCULATES CHARGED PARTICLE DENSITY AND                    CHA0C780
C               TEMPERATURE VALUES                                         CHA0C790
C      STREMR = INTRODUCES ELECTRIC FIELD PERTURBATION TO MODEL            CHA0C800
C               EFFECT OF STREAMER                                         CHA0C810
C      TRNSTY = SAME AS DENSTY EXCEPT THAT IT ALLOWS FOR A TRANSITION      CHA0C820
C               FROM LAMINAR TO TURBULENT FLOW                             CHA0C830
C      CNDUCT = COMPUTES GAMMA FROM AN INPUT                               CHA0C840
C               CHARGED PARTICLE DENSITY PROFILE                           CHA0C850
C                                                                          CHA0C860
C                                                                          CHA0C870
C  ********************************************************************    CHA0C880
C  * SET UP SUBROUTINE TO DEFINE INITIAL CONDITIONS FOR CHARGED *         CHA0C890
C  * PARTICLE DENSITY PROFILE                                   *         CHA0C900
C  ********************************************************************    CHA0C910
C                                                                          CHA0C920
      SUBROUTINE NZERO(N,RADIUS,RHATO)                                     CHA0C930
      DOUBLE PRECISION N(RADIUS,2)                                         CHA0C940
      INTEGER I,RADIUS,RHATO,RHAT1                                         CHA0C950
      RHAT1 = RHATO + 1                                                    CHA0C960
```

```
      DO 10 I = 1,RHATO                                                 CHA0C970
         N(I,1) = 1.0                                                   CHACC980
         N(I,2) = 1.0                                                   CHA0C990
   10 CONTINUE                                                          CHA01000
      DO 20 I = RHAT1,RADIUS                                            CHA01010
         N(I,1) = 0.0                                                   CHA01020
         N(I,2) = 0.0                                                   CHA01030
   20 CONTINUE                                                          CHA01040
      RETURN                                                            CHA01050
      END                                                               CHA01060
C                                                                       CHA01070
C ***************************************************************     CHA01080
C * SET UP SUBROUTINE TO DEFINE INITIAL CONDITIONS FOR TEMPERATURE *     CHA01090
C * PROFILE                                                     *     CHA01100
C ***************************************************************     CHA01110
C                                                                       CHA01120
      SUBROUTINE TZERO(T,RADIUS)                                        CHA01130
      DOUBLE PRECISION T(RADIUS,2)                                      CHA01140
      INTEGER I,RADIUS                                                  CHA01150
      DO 25 I = 1,RADIUS                                                CHA01160
         T(I,1) = 1.0                                                   CHA01170
         T(I,2) = 1.0                                                   CHA01180
   25 CONTINUE                                                          CHA01190
      RETURN                                                            CHA01200
      END                                                               CHA01210
C                                                                       CHA01220
C ***************************************************************     CHA01230
C * SET UP SUBROUTINE TO SWITCH COMPUTED DENSITY AND TEMPERATURE  *     CHA01240
C * VALUES FROM SECOND ROW BACK TO FIRST ROW FOR NEXT CALCULATION *     CHA01250
C * SEQUENCE                                                    *     CHA01260
C ***************************************************************     CHA01270
C                                                                       CHA01280
      SUBROUTINE SWITCH(N,T,RADIUS)                                     CHA01290
      DOUBLE PRECISION N(RADIUS,2),T(RADIUS,2)                          CHA01300
      INTEGER I,RADIUS                                                  CHA01310
      DO 30 I = 1,RADIUS                                                CHA01320
         N(I,1) = N(I,2)                                                CHA01330
         T(I,1) = T(I,2)                                                CHA01340
   30 CONTINUE                                                          CHA01350
      RETURN                                                            CHA01360
      END                                                               CHA01370
C                                                                       CHA01380
C ***************************************************************     CHA01390
C * SET UP SUBROUTINE TO CALCULATE CHARGED PARTICLE DENSITY VALUES *     CHA01400
C * AND TEMPERATURE PROFILE FOR ANY ARBITRARY BETA AND/OR BETAA   *     CHA01410
C * VALUE                                                       *     CHA01420
C ***************************************************************     CHA01430
C                                                                       CHA01440
```

```
      SUBROUTINE CENSTY(TIME,RADIUS,T,N,BETA,BETAA,N1)                  CHA01450
      DOUBLE PRECISION A,B,C,N(RADIUS,2),NO,A2,A3,DT,DR,ALPHA,BETA,R,   CHA01460
     *T(RADIUS,2),BETAA,D,E,DTA,N1(RADIUS,2)                            CHA01470
      INTEGER I,J,K,TIME,RADIUS,TEMRAD                                  CHA01480
      TEMRAD = RADIUS - 1                                               CHA01490
      NO = 8.32CE16                                                     CHA01500
      A2 = 1.CE-13                                                      CHA01510
      A3 = 1.E-35                                                       CHA01520
      DT = 0.1                                                          CHA01530
      DTA = 1.C57E-3                                                    CHA01540
      DR = 0.1                                                          CHA01550
      ALPHA = (A3*NO)/A2                                                CHA01560
      DO 40 K = 1,TIME                                                  CHA01570
         CALL SWITCH(N,T,RADIUS)                                        CHA01580
         DO 50 I = 2,TEMRAD                                             CHA01590
            J = 1                                                       CHA01600
            R = (FLOAT(I)-1)/10.                                        CHA01610
            A = (1./(DR*DR))*(N((I+1),J)-(2.*N(I,J))+N((I-1),J))        CHA01620
            B = (1./(2.*R*DR))*(N((I+1),J)-N((I-1),J))                  CHA01630
C           C = N(I,J)*N(I,J)+ALPHA*N(I,J)**3.                          CHA01640
C                                                                       CHA01650
C  ***********************************************************          CHA01660
C  * USE FOLLOWING VALUE FOR 'C' IF IONIZATION TERM DESIRED *          CHA01670
C  ***********************************************************          CHA01680
C                                                                       CHA01690
            C = N(I,J)*N(I,J)+ALPHA*N(I,J)**3.-N(I,J)                   CHA01700
            D = (1./(DR*DR))*(T((I+1),J)-(2.*T(I,J))+T((I-1),J))        CHA01710
            E = (1./(2.*R*DR))*(T((I+1),J)-T((I-1),J))                  CHA01720
            N(I,(J+1)) = (DT*(((1./BETA)*(A+B))-C))+N(I,J)              CHA01730
            T(I,(J+1)) = DTA*((1./BETAA)*(D+E)+N(I,J))+T(I,J)           CHA01740
            IF (N(I,(J+1)).GT.1.E-14) GO TO 54                          CHA01750
               N(I,(J+1)) = 0.0                                         CHA01760
   54       CONTINUE                                                    CHA01770
            IF (I.NE.2) GO TO 55                                        CHA01780
               N((I-1),(J+1)) = N(I,(J+1))                              CHA01790
               T((I-1),(J+1)) = T(I,(J+1))                              CHA01800
   55       CONTINUE                                                    CHA01810
            IF (I.NE.TEMRAD) GO TO 56                                   CHA01820
               T((I+1),(J+1)) = 1.0                                     CHA01830
   56       CONTINUE                                                    CHA01840
   50    CONTINUE                                                       CHA01850
   40 CONTINUE                                                          CHA01860
      RETURN                                                            CHA01870
      END                                                               CHA01880
C                                                                       CHA01890
C  ***********************************************************          CHA01900
C  * SET UP SUBROUTINE TO CALCULATE CHARGED PARTICLE DENSITY VALUES *   CHA01910
C  * AND TEMPERATURE PROFILE FOR ANY ARBITRARY BETA AND/OR BETAA    *   CHA01920
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C * VALUE DUE TO PERTURBATION OF ELECTRIC FIELD BY STREAMER          *     CHA01930
C ***************************************************************     CHA01940
C                                                                       CHA01950
      SUBROUTINE STREMR(TIME,RADIUS,T,N,BETA,BETAA,N1,Z,ZA)             CHA01960
      DOUBLE PRECISION A,B,C,N(RADIUS,2),NO,A2,A3,DT,DR,ALPHA,BETA,R,   CHA01970
     *T(RADIUS,2),BETAA,D,E,DTA,N1(RADIUS,2),Z(RADIUS),ZA(RADIUS)       CHA01980
      INTEGER I,J,K,TIME,RADIUS,TEMRAD                                  CHA01990
      TEMRAD = RADIUS - 1                                               CHA02000
      NO = 8.320E16                                                     CHA02010
      A2 = 1.E-13                                                       CHA02020
      A3 = 1.E-35                                                       CHA02030
      DT = 0.1                                                          CHA02040
      DTA = 1.057E-3                                                    CHA02050
      DR = 0.1                                                          CHA02060
      ALPHA = (A3*NO)/A2                                                CHA02070
      DO 60 K = 1,TIME                                                  CHA02080
         CALL SWITCH(N,T,RADIUS)                                        CHA02090
         DO 70 I = 2,TEMRAD                                             CHA02100
            J = 1                                                       CHA02110
            R = (FLOAT(I)-1)/10.                                        CHA02120
            A = (1./(DR*DR))*(N((I+1),J)-(2.*N(I,J))+N((I-1),J))        CHA02130
            B = (1./(2.*R*DR))*(N((I+1),J)-N((I-1),J))                  CHA02140
C           C = N(I,J)*N(I,J)+ALPHA*N(I,J)**3.                          CHA02150
C                                                                       CHA02160
C ***************************************************************     CHA02170
C * USE FOLLOWING VALUE FOR 'C' IF IONIZATION TERM DESIRED          *   CHA02180
C ***************************************************************     CHA02190
C                                                                       CHA02200
            C = N(I,J)*N(I,J)+ALPHA*N(I,J)**3.-Z(I)*N(I,J)              CHA02210
            D = (1./(DR*DR))*(T((I+1),J)-(2.*T(I,J))+T((I-1),J))        CHA02220
            E = (1./(2.*R*DR))*(T((I+1),J)-T((I-1),J))                  CHA02230
            N1(I,(J+1)) = (DT*(((1./BETA)*(A+B))-C))+N(I,J)             CHA02240
            T(I,(J+1)) = DTA*((1./BETAA)*(D+E)+ZA(I)*N(I,J))+T(I,J)     CHA02250
            IF (N(I,(J+1)).GT.1.E-14) GO TO 74                          CHA02260
               N(I,(J+1)) = 0.0                                         CHA02270
   74       CONTINUE                                                    CHA02280
            IF (I.NE.2) GO TO 75                                        CHA02290
               N((I-1),(J+1)) = N(I,(J+1))                              CHA02300
               T((I-1),(J+1)) = T(I,(J+1))                              CHA02310
   75       CONTINUE                                                    CHA02320
            IF (I.NE.TEMRAD) GO TO 76                                   CHA02330
               T((I+1),(J+1)) = T(I,(J+1))                              CHA02340
   76       CONTINUE                                                    CHA02350
   70    CONTINUE                                                       CHA02360
   60 CONTINUE                                                          CHA02370
      RETURN                                                            CHA02380
      END                                                               CHA02390
C                                                                       CHA02400
```

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C ***********************************************************
C * SET UP SUBROUTINE TO COMPUTE CHARGED PARTICLE DENSITY VALUES *
C * AND/OR TEMPERATURE PROFILES FOR TRANSITION PHASE           *
C ***********************************************************
C
      SUBROUTINE TRNSTY(TIME,RADIUS,T,N,BMIN,BMAX,BAMIN,BAMAX,N1)
      DOUBLE PRECISION A,B,C,N(RADIUS,2),NO,A2,A3,DT,DR,ALPHA,BMIN,
     *BMAX,BETA,R,T(RADIUS,2),BETAA,D,E,DTA,BAMIN,BAMAX,N1(RADIUS,2)
      INTEGER I,J,K,TIME,RADIUS,TEMRAD
      TEMRAD = RADIUS - 1
      NO = 8.320E16
      A2 = 1.0E-13
      A3 = 1.E-35
      DT = 0.1
      DTA = 1.057E-3
      DR = 0.1
      ALPHA = (A3*NO)/A2
      DO 80 K = 1,TIME
         CALL SWITCH(N,T,RADIUS)
         DO 81 I = 2,TEMRAD
            J = 1
            BETA = BMAX-(BMAX-BMIN)*(FLOAT(K)-1.)/(FLOAT(TIME)-1.)
            BETAA = BAMAX-(BAMAX-BAMIN)*(FLOAT(K)-1.)/(FLOAT(TIME)-1.)
            R = (FLOAT(I)-1.)/10.
            A = (1./(DR*DR))*(N((I+1),J)-(2.*N(I,J))+N((I-1),J))
            B = (1./(2.*R*DR))*(N((I+1),J)-N((I-1),J))
C           C = N(I,J)*N(I,J)+ALPHA*N(I,J)**3.
C
C ***********************************************************
C * USE FOLLOWING VALUE FOR 'C' IF IONIZATION TERM DESIRED *
C ***********************************************************
C
            C = N(I,J)*N(I,J)+ALPHA*N(I,J)**3.-N(I,J)
            D = (1./(DR*DR))*(T((I+1),J)-(2.*T(I,J))+T((I-1),J))
            E = (1./(2.*R*DR))*(T((I+1),J)-T((I-1),J))
            N(I,(J+1)) = (DT*(((1./BETA)*(A+B))-C))+N(I,J)
            T(I,(J+1)) = DTA*((1./BETAA)*(D+E)+N(I,J))+T(I,J)
            IF (N(I,(J+1)).GT.1.E-14) GO TO 84
               N(I,(J+1)) = 0.0
   84       CONTINUE
            IF (I.NE.2) GO TO 85
               N((I-1),(J+1)) = N(I,(J+1))
               T((I-1),(J+1)) = T(I,(J+1))
   85       CONTINUE
            IF (I.NE.TEMRAD) GO TO 86
               T((I+1),(J+1)) = T(I,(J+1))
   86       CONTINUE
   81    CONTINUE
```

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   8C CONTINUE                                                                  CHA02890
      RETURN                                                                    CHA02900
      END                                                                       CHA02910
C                                                                               CHA02920
C ***************************************************************             CHA02930
C * SET UP SUBROUTINE TO COMPUTE THE CONDUCTANCE PARAMETER *                   CHA02940
C * GAMMA BY INTEGRATION                                   *                   CHA02950
C ***************************************************************             CHA02960
C                                                                               CHA02970
      SUBROUTINE CNDUCT(RADIUS,NN,VALX,R1)                                      CHA02980
      REAL NN,VALX(RADIUS),R1(RADIUS),SUM,DR,PI                                 CHA02990
      INTEGER J,RADIUS                                                          CHA03000
      DR = 0.1                                                                  CHA03010
      NN = 0.0                                                                  CHA03020
      PI = 3.14159265 4                                                         CHA03030
      DO 90 J = 2,RADIUS                                                        CHA03040
         SUM = PI*((R1(J)*VALX(J))+(R1(J-1)*VALX(J-1)))*DR                      CHA03050
         NN = NN + SUM                                                          CHA03060
   9C CONTINUE                                                                  CHA03070
      RETURN                                                                    CHA03080
      END                                                                       CHA03090
C                                                                               CHA03100
C ************************                                                      CHA03110
C * MAIN PROGRAM BEGINS *                                                      CHA03120
C ************************                                                      CHA03130
C                                                                               CHA03140
      REAL RR(101),VALO(101),VAL1(101),VAL2(101),VAL3(101),VAL4(101),           CHA03150
     *VALTO(101),VALT1(101),VALT2(101),VALT3(101),VALT4(101)                    CHA03160
      DOUBLE PRECISION N(101,2),BETA,BMIN,BMAX,T(101,2),BETAA,                  CHA03170
     *N1(101,2),BAMIN,BAMAX,Z(101),ZA(101)                                      CHA03180
      REAL NN,R1(101),NN1,NN2,NN3,NN4,EQ                                        CHA03190
      INTEGER I,TIME,RADIUS,CONRAD,RHATO,RHAT1,RSTRMO,RSTRM1                    CHA03200
C                                                                               CHA03210
C ************************                                                      CHA03220
C * INITIALIZE VALUES *                                                        CHA03230
C ************************                                                      CHA03240
C                                                                               CHA03250
      BETA = 66.560000                                                          CHA03260
      BMIN = 23.6300                                                            CHA03270
      BMAX = 66.5600                                                            CHA03280
      BETAA = 0.62520                                                           CHA03290
      BAMIN = 0.22190                                                           CHA03300
      BAMAX = 0.62520                                                           CHA03310
      EQ = 1.4685E5                                                             CHA03320
      RADIUS = 101                                                              CHA03330
      RHATO = 11                                                                CHA03340
      RHAT1 = RHATO + 1                                                         CHA03350
      RSTRMO = 11                                                               CHA03360
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      RSTRM1 = RSTRMO + 1                                                CHA03270
      CONRAD = 11                                                        CHA03280
      DO  50  I = 1,RSTRMO                                               CHA03290
         Z(I) = 10.0                                                     CHA03300
         ZA(I) = 2.5                                                     CHA03310
   50 CONTINUE                                                           CHA03320
      DO 60 I = RSTRM1,RADIUS                                            CHA03330
         Z(I) = 1.0                                                      CHA03340
         ZA(I) = 1.0                                                     CHA03350
   60 CONTINUE                                                           CHA03360
      DO 100 I = 1,RHATO                                                 CHA03370
         N(I,1) = 1.0                                                    CHA03380
         N(I,2) = 1.0                                                    CHA03390
  100 CONTINUE                                                           CHA03400
      DO 110 I = RHAT1,RADIUS                                            CHA03410
         N(I,1) = 0.0                                                    CHA03420
         N(I,2) = 0.0                                                    CHA03430
  110 CONTINUE                                                           CHA03440
      DO 120 I = 1,RADIUS                                                CHA03450
         T(I,1) = 1.0                                                    CHA03460
         T(I,2) = 1.0                                                    CHA03470
  120 CONTINUE                                                           CHA03480
C     N1(1,1) = 1.00000                                                  CHA03490
C     N1(2,1) = 0.99750                                                  CHA03500
C     N1(3,1) = 0.99376                                                  CHA03510
C     N1(4,1) = 0.99002                                                  CHA03520
C     N1(5,1) = 0.98382                                                  CHA03530
C     N1(6,1) = 0.97762                                                  CHA03540
C     N1(7,1) = 0.96901                                                  CHA03550
C     N1(8,1) = 0.96039                                                  CHA03560
C     N1(9,1) = 0.94943                                                  CHA03570
C     N1(10,1) = 0.93846                                                 CHA03580
C     N1(11,1) = 0.92523                                                 CHA03590
C     N1(12,1) = 0.91200                                                 CHA03600
C     N1(13,1) = 0.89660                                                 CHA03610
C     N1(14,1) = 0.88120                                                 CHA03620
C     N1(15,1) = 0.86374                                                 CHA03630
C     N1(16,1) = 0.84628                                                 CHA03640
C     N1(17,1) = 0.82690                                                 CHA03650
C     N1(18,1) = 0.80752                                                 CHA03660
C     N1(19,1) = 0.78636                                                 CHA03670
C     N1(20,1) = 0.76519                                                 CHA03680
C     N1(21,1) = 0.74241                                                 CHA03690
C     N1(22,1) = 0.71962                                                 CHA03700
C     N1(23,1) = 0.69538                                                 CHA03710
C     N1(24,1) = 0.67113                                                 CHA03720
C     N1(25,1) = 0.64561                                                 CHA03730
C     N1(26,1) = 0.62008                                                 CHA03740
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C      N1(27,1) = C.59347                                                CHA03850
C      N1(28,1) = C.56685                                                CHA03860
C      N1(29,1) = C.53934                                                CHA03870
C      N1(30,1) = C.51182                                                CHA03880
C      N1(31,1) = C.48361                                                CHA03890
C      N1(32,1) = C.45540                                                CHA03900
C      N1(33,1) = C.42669                                                CHA03910
C      N1(34,1) = C.39758                                                CHA03920
C      N1(35,1) = C.36898                                                CHA03930
C      N1(36,1) = C.33998                                                CHA03940
C      N1(37,1) = C.31090                                                CHA03950
C      N1(38,1) = C.28181                                                CHA03960
C      N1(39,1) = C.25285                                                CHA03970
C      N1(40,1) = C.22389                                                CHA03980
C      N1(41,1) = C.19525                                                CHA03990
C      N1(42,1) = C.16660                                                CHA04000
C      N1(43,1) = 0.13848                                                CHA04010
C      N1(44,1) = C.11036                                                CHA04020
C      N1(45,1) = C.08295                                                CHA04030
C      N1(46,1) = C.05553                                                CHA04040
C      N1(47,1) = C.02902                                                CHA04050
C      N1(48,1) = C.00250                                                CHA04060
C      DO 210 I = 1,48                                                   CHA04070
C         N1(I,2) = N1(I,1)                                              CHA04080
C 21C CONTINUE                                                           CHA04090
C      DO 220 I = 49,RADIUS                                              CHA04100
C         N1(I,1) = 0.0                                                  CHA04110
C         N1(I,2) = 0.0                                                  CHA04120
C 22C CONTINUE                                                           CHA04130
C                                                                        CHA04140
C ********************************************************************** CHA04150
C * COMPUTE VARIOUS CHARGED PARTICLE DENSITY AND TEMPERATURE PROFILES * CHA04160
C * AS NEEDED                                                        * CHA04170
C ********************************************************************** CHA04180
C                                                                        CHA04190
      DO 300 I = 1,RADIUS                                                CHA04200
         VALC(I) = N(I,1) + 1.E-70                                       CHA04210
  30C CONTINUE                                                           CHA04220
C     DO 310 I = 1,RADIUS                                                CHA04230
C        VALC(I) = N1(I,1) + 1.E-70                                      CHA04240
C        N(I,1) = N1(I,1)                                                CHA04250
C        N(I,2) = N1(I,1)                                                CHA04260
C 31C CONTINUE                                                           CHA04270
      DO 350 I = 1,RADIUS                                                CHA04280
         VALT(I) = T(I,1)                                                CHA04290
  35C CONTINUE                                                           CHA04300
C     CALL DENSTY(0001001,RADIUS,T,N,BETA,BETAA,N1)                      CHA04310
C     CALL STREMR(000000,RADIUS,T,N,BETA,BETAA,N1,Z,ZA)                  CHA04320
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      CALL TRNSTY(000101,RADIUS,T,N,BMIN,EMAX,BAMIN,BAMAX,N1)          CHA04330
      CALL DENSTY(024900,RADIUS,T,N,BMIN,BAMIN,N1)                     CHA04340
      DO 400 I = 1,RADIUS                                              CHA04350
C        VAL1(I) = N(I,1) + 1.E-70                                     CHA04360
         VALT1(I) = T(I,1) + 1.E-70                                    CHA04370
  40C CONTINUE                                                         CHA04380
C     CALL NZERO(N,RADIUS,RHATO)                                       CHA04390
C     CALL TZERO(T,RADIUS)                                             CHA04400
C     CALL DENSTY(005001,RADIUS,T,N,BMIN,BAMIN,N1)                     CHA04410
      CALL STREMR(000500,RADIUS,T,N,BETA,BETAA,N1,Z,ZA)                CHA04420
C     CALL TRNSTY(009000,RADIUS,T,N,5000.,BMAX,BAMIN,BAMAX,N1)         CHA04430
C     CALL DENSTY(010000,RADIUS,T,N,5000.,BAMIN,N1)                    CHA04440
      DO 410 I = 1,RADIUS                                              CHA04450
C        VAL2(I) = N(I,1) + 1.E-70                                     CHA04460
         VALT2(I) = T(I,1) + 1.E-70                                    CHA04470
  41C CONTINUE                                                         CHA04480
C     CALL NZERO(N,RADIUS,RHATO)                                       CHA04490
C     CALL TZERO(T,RADIUS)                                             CHA04500
C     CALL DENSTY(005001,RADIUS,T,N,BMIN,BAMIN,N1)                     CHA04510
C     CALL STREMR(000100,RADIUS,T,N,BETA,BETAA,N1,Z,ZA)                CHA04520
      CALL TRNSTY(000100,RADIUS,T,N,BMIN,EMAX,BAMIN,BAMAX,N1)          CHA04530
      CALL DENSTY(00C100,RADIUS,T,N,BMIN,BAMIN,N1)                     CHA04540
      DO 420 I = 1,RADIUS                                              CHA04550
C        VAL3(I) = N(I,1) + 1.E-70                                     CHA04560
         VALT3(I) = T(I,1) + 1.E-70                                    CHA04570
  42C CONTINUE                                                         CHA04580
C     CALL NZERO(N,RADIUS,RHATO)                                       CHA04590
C     CALL TZERO(T,RADIUS)                                             CHA04600
      CALL DENSTY(001000,RADIUS,T,N,BMIN,BAMIN,N1)                     CHA04610
C     CALL STREMR(000100,RADIUS,T,N,BETA,BETAA,N1,Z,ZA)                CHA04620
C     CALL TRNSTY(009000,RADIUS,T,N,50.0,EMAX,BAMIN,BAMAX,N1)         CHA04630
C     CALL DENSTY(010000,RADIUS,T,N,50.0,BETAA,N1)                     CHA04640
      DO 430 I = 1,RADIUS                                              CHA04650
C        VAL4(I) = N(I,1) + 1.E-70                                     CHA04660
         VALT4(I) = T(I,1) + 1.E-70                                    CHA04670
  43C CONTINUE                                                         CHA04680
C                                                                      CHA04690
C ******************************************************               CHA04700
C * PERFORM INTEGRATIONS FOR CONDUCTANCE CALCULATIONS *                CHA04710
C ******************************************************               CHA04720
C                                                                      CHA04730
C     DO 499 I = 1,RADIUS                                              CHA04740
C        R1(I) = (FLOAT(I)-1.)/10.                                     CHA04750
C 499 CONTINUE                                                         CHA04760
C     CALL CNDUCT(RADIUS,NN,VAL1,R1)                                   CHA04770
C     NN1 = NN                                                         CHA04780
C     CALL CNDUCT(RADIUS,NN,VAL2,R1)                                   CHA04790
C     NN2 = NN                                                         CHA04800
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C      CALL CNELCT(RADIUS,NN,VAL3,R1)                                    CHA04E10
C      NN3 = NN                                                          CHA04E20
C      CALL CNELCT(RADIUS,NN,VAL4,R1)                                    CHA04E30
C      NN4 = NN                                                          CHA04E40
C                                                                        CHA04E50
C  ***************                                                       CHA04E60
C  * PRINT DATA *                                                       CHA04E70
C  ***************                                                       CHA04E80
C                                                                        CHA04E90
C      WRITE(6,500)BETA                                                  CHA04900
C  500 FORMAT(2X,'FOR 100TR AND BETA =',1X,F8.1)                         CHA04910
C      WRITE(6,510)                                                      CHA04920
C  510 FORMAT(2X,'FOR 4000TR AND')                                       CHA04930
C      WRITE(6,550)NN1                                                   CHA04940
C      WRITE(6,551)NN2                                                   CHA04950
C      WRITE(6,552)NN3                                                   CHA04960
C      WRITE(6,553)NN4                                                   CHA04970
C  550 FORMAT(2X,'BETA = 50; CONDUCTANCE =',1X,E15.6)                    CHA04980
C  551 FORMAT(2X,'BETA = 500; CONDUCTANCE = ',1X,E15.6)                  CHA04990
C  552 FORMAT(2X,'BETA = 5000; CONDUCTANCE = ',1X,E15.6)                 CHA05000
C  553 FORMAT(2X,'BETA = 50000; CONDUCTANCE = ',1X,E15.6)                CHA05010
C      WRITE(6,560)N1                                                    CHA05020
C  560 FORMAT(6(1X,F10.6))                                               CHA05030
C      WRITE(6,600)VAL0                                                  CHA05040
C      WRITE(6,600)VAL1                                                  CHA05050
C      WRITE(6,600)VAL2                                                  CHA05060
C      WRITE(6,600)VAL3                                                  CHA05070
C      WRITE(6,600)VAL4                                                  CHA05080
       WRITE(6,600)VALT0                                                 CHA05090
       WRITE(6,600)VALT1                                                 CHA05100
       WRITE(6,600)VALT2                                                 CHA05110
       WRITE(6,600)VALT3                                                 CHA05120
       WRITE(6,600)VALT4                                                 CHA05130
  600 FORMAT(5(1X,F15.6))                                                CHA05140
C                                                                        CHA05150
C  ****************                                                      CHA05160
C  * PLOT RESULTS *                                                     CHA05170
C  ****************                                                      CHA05180
C                                                                        CHA05190
       DO 700 I = 1,RADIUS                                               CHA05200
          RR(I) = (FLOAT(I)-1.0)/10.0                                    CHA05210
  700 CONTINUE                                                           CHA05220
       CALL COMPRS                                                       CHA05230
C      CALL TEK618                                                       CHA05240
       CALL NCCHEK                                                       CHA05250
       CALL PAGE(8.5,11.)                                                CHA05260
       CALL PHYSCR(1.75,2.5)                                             CHA05270
       CALL AREA2D(5.,6.)                                                CHA05280
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C       CALL HEADIN('CHARGED PARTICLE DENSITY PROFILE$',100,-2,2)          CHA05290
        CALL HEADIN('TEMPERATURE PROFILE$',100,-2,2)                       CHA05300
C       CALL HEADIN('IN TURBULENT FLOW$',100,-2,2)                         CHA05310
        CALL HEADIN('WITH STREAMER ON 50 TR$',100,-2,2)                    CHA05320
C       CALL HEADIN('IN LAMINAR FLOW$',100,-2,2)                           CHA05330
C       CALL HEADIN('FOR BAMIN = 800$',100,-2,2)                           CHA05340
C       CALL HEADIN('BESSEL FUNCTION PROFILE INITIAL CONDITION$',100,-2,3)CHA05350
C       CALL HEADIN('WITH IONIZATION TERM$',100,-2,2)                      CHA05360
        CALL XNAME('R/RO$',100)                                            CHA05370
C       CALL YNAME('CHARGE DENSITY (N/NO)$',100)                           CHA05380
        CALL YNAME('TEMPERATURE (T/TO)$',100)                              CHA05390
        CALL GRAF(0.0,1.00,10.0,0.000,01.0,14.00)                          CHA05400
        CALL DCT                                                           CHA05410
        CALL GRID(1,1)                                                     CHA05420
        CALL RESET('DOT')                                                  CHA05430
C       CALL YLOG(0.0,2.000,1.E-05,0.600)                                  CHA05440
C       CALL MESSAG('BESSEL FUNCTION$',100,2.8,5.6)                        CHA05450
C       CALL MESSAG('INITIAL CONDITION$',100,2.8,5.4)                      CHA05460
        CALL MESSAG('EO = 1.47E5 V/M$',100,2.8,5.4)                        CHA05470
C       CALL MESSAG('BETA = $',100,3.5,5.8)                                CHA05480
C       CALL REALNO(BETA,-2,4.0,5.8)                                       CHA05490
C       CALL MESSAG('BETAA = $',100,4.,5.6)                                CHA05500
C       CALL REALNO(BETAA,1,4.8,5.6)                                       CHA05510
C       CALL MESSAG('EO = $',100,4.,5.4)                                   CHA05520
C       CALL REALNO(EO,-2,5.0,5.4)                                         CHA05530
C       CALL MESSAG('Z = $',100,4.,5.2)                                    CHA05540
C       CALL REALNO(Z,2,4.8,5.2)                                           CHA05550
C       CALL MESSAG('ZA = $',100,4.,5.0)                                   CHA05560
C       CALL REALNO(ZA,2,4.8,5.0)                                          CHA05570
        CALL MESSAG('FIGURE 3.28$',100,2.0,-1.0C)                          CHA05580
        CALL GFACE(0)                                                      CHA05590
C       CALL CURVE(RR,VAL0,101,5)                                          CHA05600
C       CALL DASH                                                          CHA05610
C       CALL CURVE(RR,VAL1,101,5)                                          CHA05620
C       CALL RESET('DASH')                                                 CHA05630
C       CALL DASH                                                          CHA05640
C       CALL CURVE(RR,VAL2,101,5)                                          CHA05650
C       CALL RESET('DASH')                                                 CHA05660
C       CALL DASH                                                          CHA05670
C       CALL CURVE(RR,VAL3,101,5)                                          CHA05680
C       CALL RESET('DASH')                                                 CHA05690
C       CALL DASH                                                          CHA05700
C       CALL CURVE(RR,VAL4,101,5)                                          CHA05710
C       CALL RESET('DASH')                                                 CHA05720
        CALL CURVE(RR,VALT0,101,5)                                         CHA05730
C       CALL DASH                                                          CHA05740
        CALL CURVE(RR,VALT1,101,5)                                         CHA05750
C       CALL RESET('DASH')                                                 CHA05760
```

```
C       CALL DASH                                                       CHA05770
        CALL CURVE(RR,VALT2,101,5)                                      CHA05780
C       CALL RESET('DASH')                                              CHA05790
        CALL DASH                                                       CHA05800
        CALL CURVE(RR,VALT3,101,5)                                      CHA05810
C       CALL RESET('DASH')                                              CHA05820
C       CALL DASH                                                       CHA05830
        CALL CURVE(RR,VALT4,101,5)                                      CHA05840
        CALL RESET('DASH')                                              CHA05850
        CALL LINES('TIME ZERO$',LP,1)                                   CHA05860
        CALL LINES('2500 TR$',LP,2)                                     CHA05870
        CALL LINES('2550 TR$',LP,3)                                     CHA05880
        CALL LINES('2570 TR$',LP,4)                                     CHA05890
        CALL LINES('2670 TR$',LP,5)                                     CHA05900
        CALL LEGEND(LP,5,2.8,4.0)                                       CHA05910
        CALL ENDPL(0)                                                   CHA05920
        CALL DONEPL                                                     CHA05930
        STOP                                                            CHA05940
        END                                                             CHA05950
```

APPENDIX B

HP-41 PROGRAM

In the course of developing the results of Chapter III, it is necessary to perform a large number of simple but tedious calculations. The following HP-41CV program proved helpful. It should be easily adaptable to any small programmable calculator.

The program prompts the user for E/N ($V\text{-}cm^2$), ν_i/N (cm^3/s), N (cm^{-3}), α_2 (m^3/s) and v_d (cm/s). These data are available from Figure 2.2.

The program then displays E (V/m), N_o (m^{-3}), α_2 (m^3/s), DTA, BETAL (laminar), BETAT (turbulent), BETAAL (laminar), BETAAT (turbulent) and DT in true time in microsecs. The minimum BETA values computed are consistent with the stability criterion, Equation (2.17). It is good practice to add about 1% to these values to ensure a smooth running calculation.

```
01  LBL [Name Program]
02  E/N?
03  PROMPT
04  STO01
05  V/N?
06  PROMPT
07  STO02
08  N?
09  PROMPT
10  STO03
11  ALPHA2?
12  PROMPT
13  STO04
14  Vd?
15  PROMPT
16  STO05
17  RCL01
18  ENTER
19  RCL03
20  *
21  100
22  *
23  STO06
24  RCL02
25  ENTER
26  RCL03
27  *
28  RCL04
29  /
30  STO07
31  RCL04
32  ENTER
33  RCL07
34  *
35  8E-3
36  *
37  STO08
38  RCL04
39  ENTER
40  RCL07
41  *
42  1/X
43  STO09
44  RCL09
45  ENTER
46  10
47  /
48  STO10
49  1.6E-19
50  ENTER
51  RCL07
52  *
53  RCL05
54  *
55  RCL06
56  /
57  100
58  /
59  STO11
60  2.223E5
61  ENTER
62  RCL11
63  /
64  RCL06
65  X↗2
66  /
67  STO12
68  RCL11
69  ENTER
70  RCL06
71  X↗2
72  *
73  3.199E-8
74  *
75  STO13
76  RCL10
77  ENTER
78  RCL12
79  /
80  STO14
81  200
82  ENTER
83  RCL14
84  *
85  STO15
86  RCL08
87  ENTER
88  RCL15
89  *
90  RCL13
91  /
92  STO16
93  RCL06
94  E =
95  ARCL X
96  AVIEW
97  STOP
98  RCL07
99  N0 =
100 ARCL X
101 AVIEW
102 STOP
103 RCL04
104 ALPHA2 =
105 ARCL X
106 AVIEW
107 STOP
108 RCL14
```

```
109  DTA =
110  ARCL X
111  AVIEW
112  STOP
113  RCL08
114  BETAL =
115  ARCL X
116  AVIEW
117  STOP
118  RCL16
119  BETAT =
120  ARCL X
121  AVIEW
122  STOP
123  RCL13
124  BETAAL =
125  ARCL X
126  AVIEW
127  STOP
128  RCL15
129  BETAAT =
130  ARCL X
131  AVIEW
132  STOP
133  RCL10
134  ENTER
135  1E-6
136  /
137  DTT =
138  ARCL X
139  AVIEW
140  END
```

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